DEVELOPMENT LESSONS

IN

MENSURATION

WITH OVER A THOUSAND EXAMPLES AND PROBLEMS
ILLUSTRATING THE PRACTICAL USE
OF MENSURATION

BY

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CHICAGO

PREFACE.

This little book is not intended to be a scientific treatise of the subject of Mensuration, but is designed for pupils who cannot attend school long enough to study Geometry, and still ought to understand the most practical subject in Arithmetic. Nothing is attempted here that an average pupil in the upper grades cannot understand.

In the subject of Mensuration, as well as in all things taught, one should start out right; start with correct and clear conceptions. To do this the teacher should begin with something concrete. In mensuration of surfaces, figures on the blackboard will do, but when dealing with solids the teacher should procure these forms in wood, or, what is sometimes better, cut them from an apple or a potato in the presence of the class. When pupils have a clear conception of the idea in the concrete they should be led to imagine, and later learn to reason in the abstract.

All are agreed that a clear definition of terms is essential in the study of Geometry, but experience has taught many that the elements of many branches can be taught without any attempt at exact definitions. We human beings, in fact, make very little attempt at defining. We know a man, a hat, a house, when we see it, without ever having learned to define it. In like manner will pupils know a square, an angle and a circle without knowing the definition, if the teacher applies these terms to certain figures and represented ideas. The author therefore suggests that the teacher of this subject apply correct terms to ideas correctly represented, and not stop to develop a definition.

The chief aim of the teacher must be that each pupil has a clear conception of the step under consideration, before beginning the next, because in this subject especially does the next step always depend upon a preceding one.

WM. F. SELL.

Milwaukee, Wis., Nov. 24, 1903.

MENSURATION OF SURFACES.

LESSON I.

LINES.

1. Draw a horizontal line 4 inches long.

2. Draw a vertical line 6 inches long.

3. Draw two horizontal lines parallel to each other.

4. Draw five vertical lines parallel to each other.

5. Draw a line parallel to the left edge of your paper.

6. Draw a line parallel to the base of the blackboard.

7. Without measuring them draw lines of different lengths on slate, paper or blackboard. Estimate the length of each line. Test, and see how near you have come to the correct length.

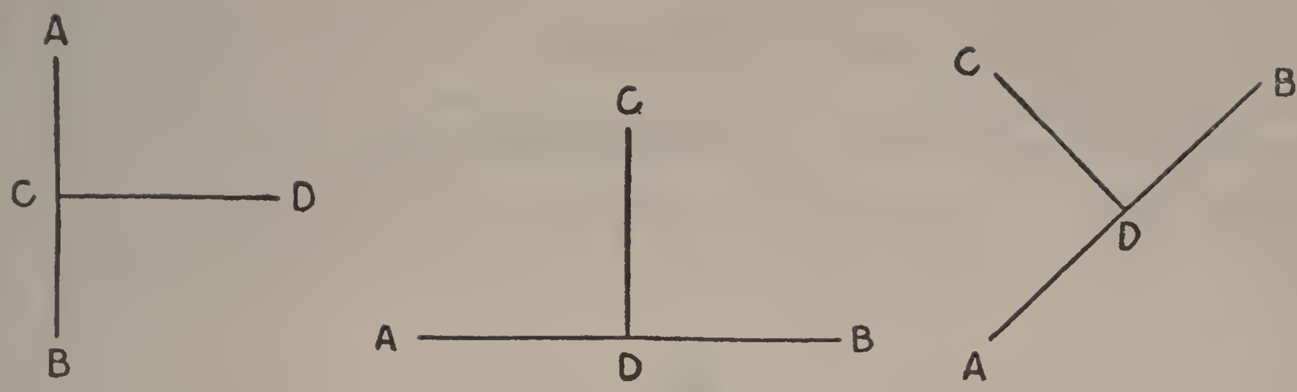
8. Estimate the length and width of your room, your desk, your school yard, the windows, the doors, the blackboard, the height of your room, the width of the road or street. Test each, and see how near you have come to it.

9. Name lines or edges that are parallel with the pieces of the window casings of your room.

10. What lines or edges are parallel with the edges of your desk?

11. Name other lines and edges in your room that are parallel to each other.

12. Draw a line perpendicular to another line.



NOTE: In each of the above three cases *c d* is perpendicular to *a b;* that is, *c d* meets *a b* so that at their point of union a square corner, or a right angle, is formed. A *vertical* line is a straight up and down line. A line is *perpendicular* to another line when it meets that line at right angles.

13. Draw a line perpendicular to the left edge of your paper.

14. Hold your pencil perpendicular to the top of your desk; to the blackboard.

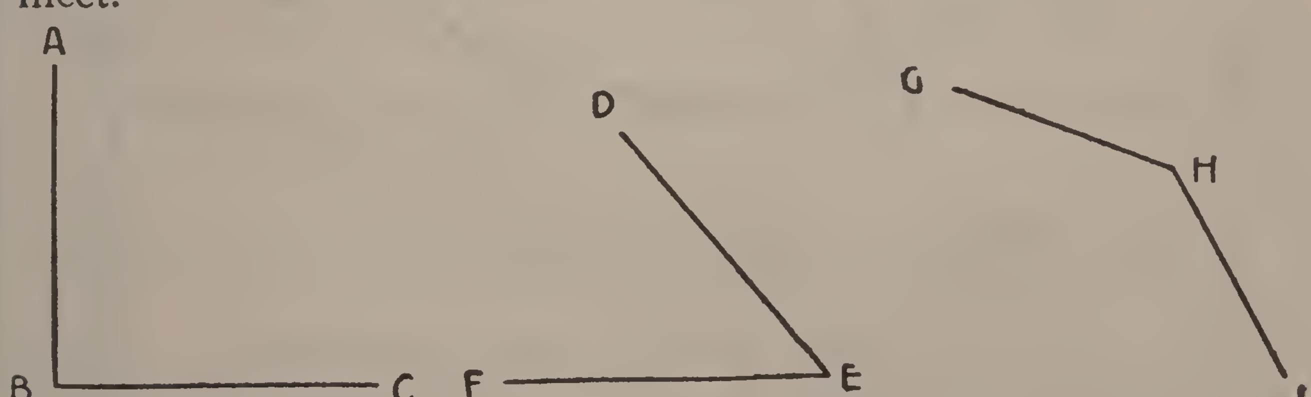
15. Tell what lines, edges and plane surfaces are perpendicular to other lines, edges and plane surfaces in your room.

16. Hold your book up in different positions and let another pupil hold his ruler perpendicular to the cover of the book.

LESSON II.

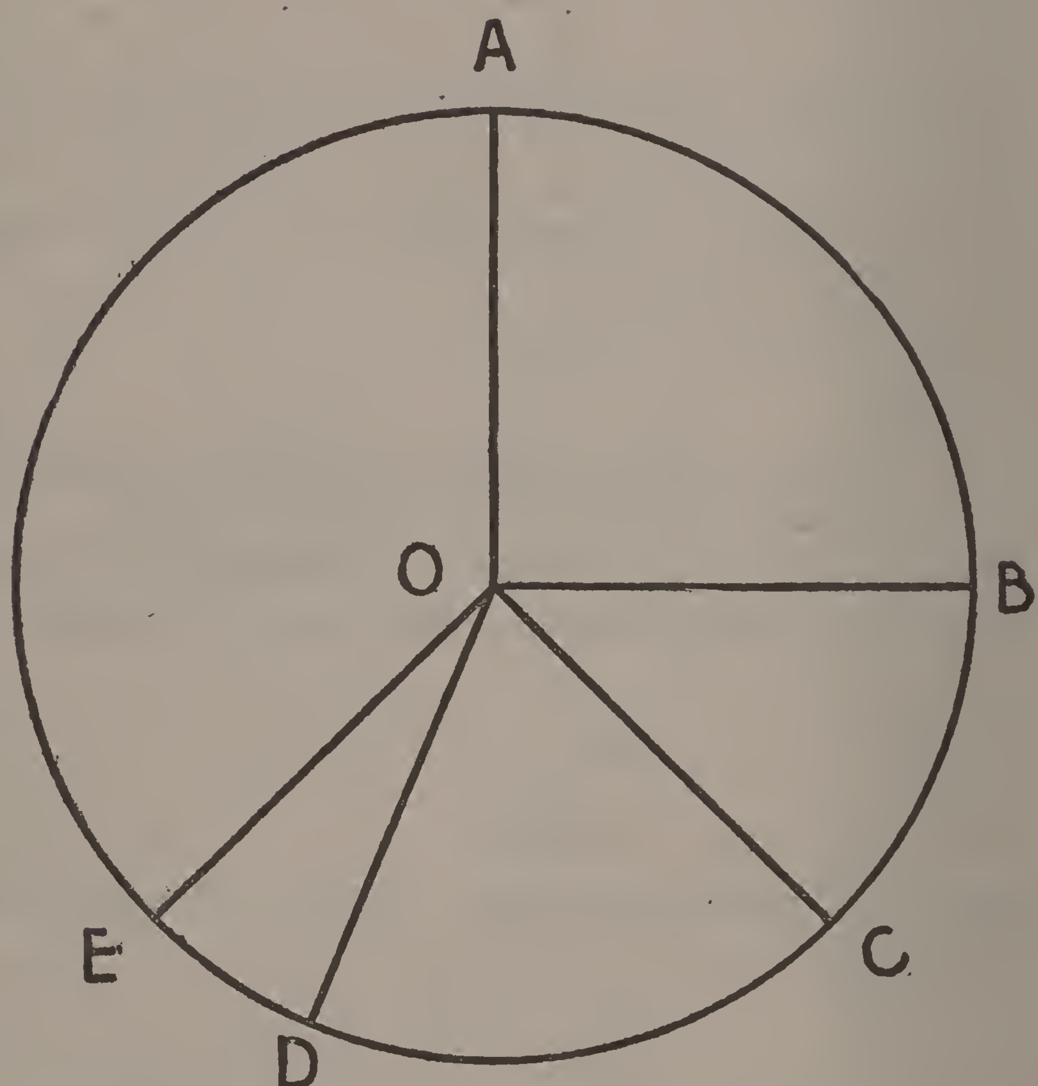
ANGLES.

An angle is a corner, or a figure made by two lines which meet.



The above figures are angles of different kinds. *a b c* forms a square corner, or angle. Such an angle is called a *right* angle.

d e f forms a sharp, or an *acute* angle. *g h i* forms a blunt, or an *obtuse* angle. When an angle is larger than a right angle, it is an obtuse angle; when it is smaller than a right angle, it is an acute angle.



1. In the above figure, what kind of an angle is *a o e*? *a o b*? *e o b*? *e o c*? *d o e*? *a o c*? *c o b*?

2. A circle is divided into 360 parts, called *degrees*. How many degrees in one half of a circle? In one quarter of a circle? In one eighth of a circle?

3. *a o b* is an angle of how many degrees?

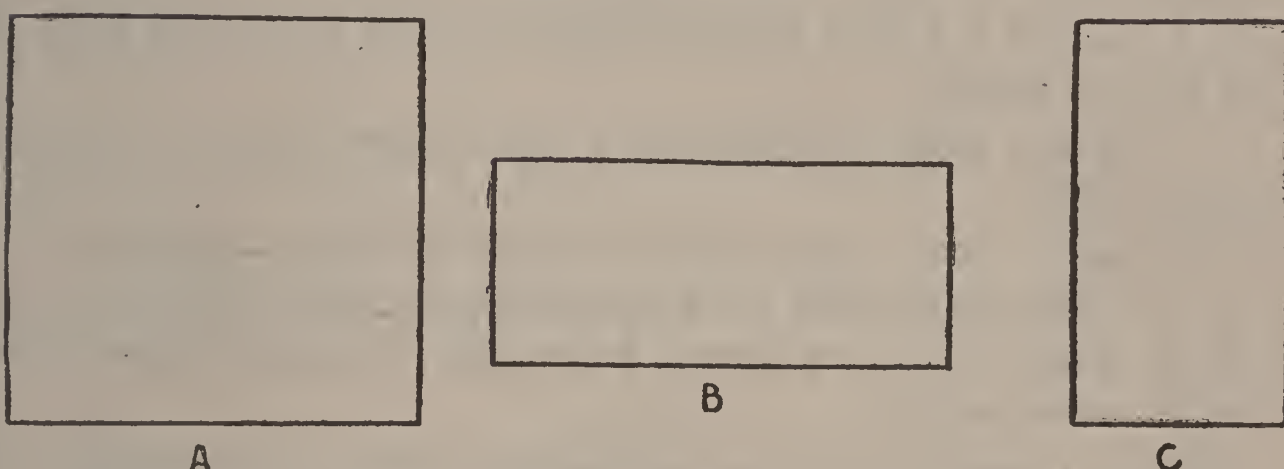
4. Of about how many degrees is the angle *b o c*? *d o c*? *e o d*? *e o a*?

5. Draw an angle of about 60 degrees. Of about 135 degrees. Of about 100 degrees. 150 degrees. 30 degrees.

6. Can you draw an angle of 180 degrees? Explain.

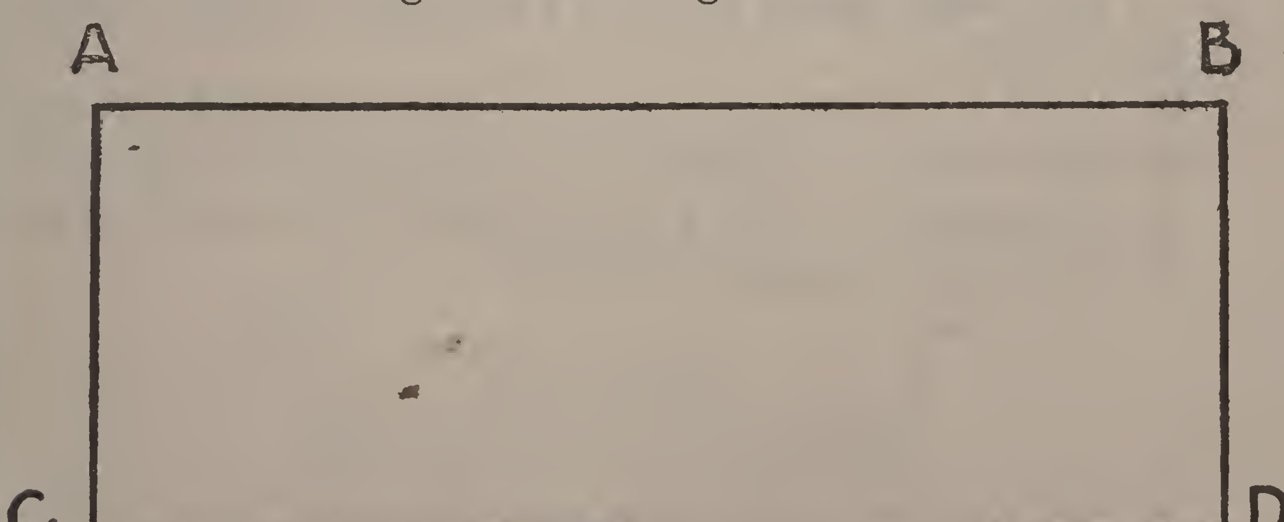
LESSON III.

RECTANGLES.



The above three figures are *rectangles* because their angles are right or *rect*angles. *a* is a square; *b* and *c* are *oblongs*.

1. What is the difference between a square and an oblong?
2. Draw a square 4 inches each way.
3. Draw a rectangle 6 inches long and 4 inches wide.



c d or *a b* is called the *base*; *a c* or *b d* is called the *altitude*.

4. In the above rectangle what can you say of the lines *a b* and *c d* as to their relation to each other?

5. What can you say as to their length?

6. If *a b* is 10 inches, how long is *c d*?

7. If *a c* is 4 inches, what is the length of *b d*?

8. Draw a rectangle 6 by 8 inches.

9. Draw one on the blackboard 2 by 3 feet.

10. Point out five rectangles in your room and estimate the dimensions of each.

11. Draw a 4 by 6 inch rectangle on your paper and letter it as the one above.

12. Cut a piece of cardboard 1 inch square. This is called a square inch.

13. How many square inches can you lay along the line *c d*?

14. How many such rows can you lay in the rectangle?

15. Then how many square inches in a rectangle 6 inches long and 4 inches wide?

NOTE: If multiplication has been taught correctly, the only logical and correct way is to say, 6 times 1 sq. in. = 6 sq. in., (as there are 6 times one square inch along the line *c d*); and, since there are four rows of 6 sq. in., 4 times 6 sq. in. = 24 sq. in. But for all practical purposes it will be enough to say, the base times the altitude, or $6 \times 4 = 24$, calling it 24 square inches.

16. How many square inches in a rectangle 10 inches long and 5 inches wide?

17. What is the area of a rectangle 6 feet long and 3 feet wide?

18. The area of a rectangle 7 by 12 feet?

19. The area of a square 8 feet on a side? One foot on a side?

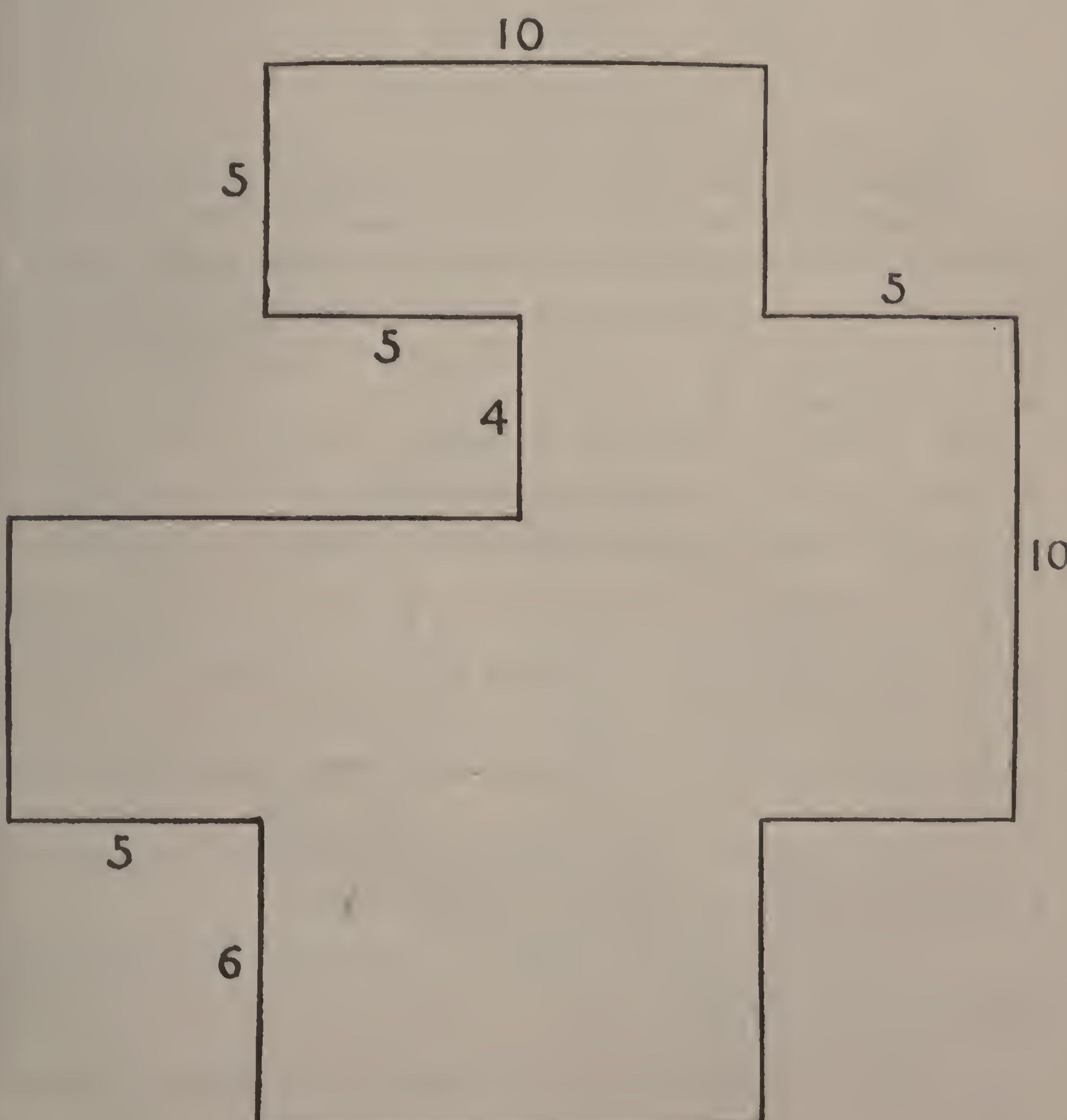
20. Area of a rectangle $16\frac{1}{2}$ by 12?

LESSON IV.

PROBLEMS IN RECTANGLES.

1. Find the area of the top of your desk.

2. The area of your school room.
3. The area of the west wall of your room.
4. The area of your blackboard.
5. The area of the sidewalk in front of your house.
6. The area of your mother's sitting-room.
7. Bring to class five other areas that you have found.
8. Find the area of the figure below.
9. Is there more than one way of finding the area? Try it.



10. What is the area of your school ground after deducting the area of the ground occupied by buildings?

11. What is the area of the four walls of your room after the area of the door and windows has been deducted?

12. How much of the four walls of your room is not covered by blackboards?

LESSON V.

LUMBER MEASURE.

A board one foot long, one foot wide and one inch, or less, thick is called a *board foot* in lumber measure. If it is thinner than one inch it is figured the same as one inch in thickness; if thicker than an inch it is figured exactly according to its thickness. A board 12 feet long, one foot wide and $1\frac{1}{4}$ inches thick contains 15 feet of lumber; $1\frac{1}{2}$ inches thick, 18 feet; $\frac{1}{2}$ inch thick, 12 feet. Usually, however, dealers do not figure a fraction of an inch in thickness, but set the price in proportion, and sell according to surface only, and in odd widths and thicknesses according to linear feet.

1. How many feet of lumber in a board 16 feet long, 1 foot wide and 1 inch thick? 2 inches thick? $1\frac{1}{4}$ inches thick? $1\frac{1}{2}$ inches thick? $1\frac{3}{4}$ inches thick? $\frac{1}{2}$ inch thick?

2. Find the number of feet of lumber in a board 18 feet long, 1 inch thick and 10 inches wide. 14 inches wide. 8 inches wide.

3. How many feet of lumber in the same boards 2 inches thick? $\frac{3}{4}$ inch thick? $1\frac{1}{2}$ inches thick? $2\frac{1}{4}$ inches thick.

4. How many feet of lumber in a scantling 12 feet long, 4 inches wide and 2 inches thick? In a joist 16 feet long, 6 inches wide and 2 inches thick?

5. Find the number of feet of lumber in the following twenty examples:

LENGTH.	WIDTH.	THICKNESS.	LENGTH.	WIDTH.	THICKNESS.
8 ft.	10 in.	1½ in.	14 ft.	4 in.	½ in.
12 ft.	6 in.	2 in.	10 ft.	8 in.	6 in.
10 ft.	14 in.	2 in.	18 ft.	10 in.	2½ in.
16 ft.	8 in.	2½ in.	16 ft.	14 in.	2 in.
14 ft.	12 in.	3 in.	12 ft.	6 in.	2 in.
18 ft.	6 in.	4 in.	14 ft.	8 in.	8 in.
8 ft.	4 in.	1¾ in.	8 ft.	4 in.	4 in.
12 ft.	11 in.	1¾ in.	6 ft.	14 in.	8 in.
16 ft.	9 in.	2 in.	8 ft.	10 in.	10 in.
14 ft.	5 in.	3 in.	20 ft.	12 in.	12 in.

6. If an inch board 1 foot wide and 16 feet long is laid on the floor, how much surface will it cover?

7. How much surface will 10 such boards cover?

8. How much surface will these boards cover if 2 inches thick? If $1\frac{1}{2}$ inches thick? Has thickness anything to do with the covering of surfaces?

LESSON VI.

KINDS OF LUMBER.

Scantlings are usually 3 or 4 inches wide, and 2, 3 or 4 inches thick, and are used upright in the frame of a building. A 4 x 4 scantling is 4 inches wide and 4 inches thick, and is used in the corner of the frame. A 2 x 4 scantling is used between the corners. Scantlings used as above are called *studding*.

Joists are usually 2 x 6, or 2 x 8, and are set on edge to support floors.

Timbers are usually 6 x 6, or larger, and are laid on the foundation of buildings, where they are called *sills*; when used in the upper part of buildings they are called *beams*.

Sidings are usually $\frac{1}{2}$ inch thick and 6 inches wide, and are laid one inch over each other on the outside of frame houses.

Planks are usually $1\frac{1}{2}$ to 3 inches thick, and are used for sidewalks, heavy floors in barns, and bridges.

Flooring is usually 1, $1\frac{1}{2}$ or 2 inches thick and 2 to 6 inches wide, and is used for floors and wainscoting.

Fence boards are usually 6 inches wide and 1 inch thick.

Rafters are scantlings used to support a roof.

Stringers are scantlings used to support a sidewalk, or for forming the horizontal parts of a tight board fence.

Laths are sold in bunches of 50, and a bunch covers 3 square yards.

Shingles are sold in bunches of a quarter thousand; 10 shingles laid 4 inches to the weather will cover one square foot.

1. How many feet of inch lumber 1 foot wide and 16 feet long will it take to cover a floor 16 feet long and 12 feet wide? How many feet of lumber will it take if boards are only 6 inches wide? If boards are 12 feet long and 12 inches wide?

NOTE: Lumber is usually cut 10, 12, 14, 16 or 18 feet in length, hence, in practical calculations, an allowance must be made for waste in covering surfaces of odd dimensions.

2. How many feet of inch lumber will it take to cover a wall 20 feet long and 10 feet high? To cover a wall 24 by 10 feet?

3. A building is 20 feet long, 16 feet wide and 10 feet high; find the dimensions of one side wall. Of one end wall.

4. How many walls are there? How many square feet in each?

5. How many feet of inch lumber will it take to cover the four walls of a building 15 by 30 feet and 9 feet high? How many feet will it take to cover the floor? The ceiling?

6. The best way to find the dimensions of the four walls of a room or building is to multiply the distance around the room or building by its height. Explain why this will give the same answer as the one obtained by finding the dimensions of each wall separately.

7. Measure your mother's sitting-room, and determine the amount of inch lumber necessary to cover wall, ceiling and floor.

8. If the roof of a building is 22 feet long and the rafters are 12 feet long, how much inch lumber will it take to cover one side of the roof? Both sides?

9. How many bunches of shingles will it take to cover that roof?

10. Find the number of bunches of laths it will take to cover the four walls of the building 18 x 20, and 10 feet high.

11. If the floor of this building is covered with $1\frac{1}{2}$ inch flooring, how much lumber will it take? If 2 inch plank is taken?

12. Lumber is sold by the thousand feet, written per M. If lumber is sold at $20 per M, find the cost of 1,500 feet.

13. Find the cost of 200 feet Of 18 feet.

14. If the lumber in example eight costs $18 per M, find cost?

15. If scantling cost $21 per M, what will fifteen 4 x 4 scantlings, 12 feet long, cost?

16. Find the cost of twenty joists, each 2 x 6, 12 feet long, at $20 per M.

17. What will the shingles in example nine cost at $3.50 per M? Remember that parts of bunches cannot be bought.

18. Find the cost of this bill of lumber:

12 2 x 4 scantlings, each 16 feet, at $16.00 per M.
10 2 x 6 joists, each 14 feet, at $18.00 per M.
5 2 x 12 planks, each 16 feet, at $20.00 per M.
3 8 x 8 timbers, each 20 feet, at $25.00 per M.

LESSON VII.

PROBLEMS IN LUMBER.

1. A barn is 40 x 80 feet and 20 feet high to the eaves. The rafters are 29 feet long. Cover it and the roof with inch boards costing $18 per M. It takes 800 feet to cover the two gable ends. Shingles cost $3.25 per M. A floor, 20 feet wide, is laid across the barn. The floor is made of 2 inch planks costing $20 per M. Find the cost of all. (Note that the floor is as long as the barn is wide.) A fraction of a board foot is figured as a whole foot.

2. A sidewalk is built of $1\frac{1}{2}$ inch lumber costing \$22 per M. The boards are nailed on two 4 x 4 stringers costing \$18 per M. What is the cost of a walk 10 rods long and 6 feet wide?

3. Find the cost of building a solid board fence 5 feet high and 120 feet long. The boards cost \$20 per M and the two 2 x 4 stringers upon which the boards are nailed cost the same per M. The posts cost 20 cents apiece, and they are set 8 feet apart.

4. What is the length of the boards you are going to buy for this fence? What length of scantlings will you buy for stringers? Think carefully before you decide upon the number of posts. What length of boards will you buy for the walk in example two?

5. An open board fence, made of 6 inch fence boards, 6 boards high, is built around a lot 70 x 140 feet. The posts are set 7 feet apart and cost 22 cents apiece. The lumber costs \$18 per M. Find cost. What length fence boards will you buy? Why are stringers not necessary here? Never carry any more than three decimal places. A fraction of a cent in the total is called a cent.

6. Build a walk, 6 feet wide, around the outside of this lot, using lumber and prices as given in example two. Find cost.

7. Build a similar walk around the inside of this lot and find cost.

8. Find the cost of building a shed 20 x 40 feet and 10 feet high to the eaves. Let all lumber used in it cost \$20 per M and shingles \$3 per M. As studding, use 4 x 4's on each corner, and 2 x 4's every two feet between. Rafters are of 2 x 4's, 15 feet long, and two feet apart. Use 6 x 6 timbers as sills, and as plates for the rafters to rest on, nail two 2 x 4's on top of each other. It takes as much board and 2 x 4 scantling for the two gable ends as it does for one end of the shed. 2 x 8 joists two feet apart are used to support the floor. The floor is made of 2 inch plank.

9. What length scantlings are you going to buy for the rafters? For the studding? What length joists should you buy? Figure carefully the number of scantlings you need for studding. Number of joists. Number of rafters.

LESSON VIII.

PROBLEMS IN SURFACE MEASURE.

1. A school room 24 x 36 feet and 15 feet high, has eight 3 x 6 foot windows in it, each 3 feet from the floor, and two 4 x 7 doors. It has 3 feet of wainscoting all around and a 3 foot wide slate blackboard all around. Slate board is set over the plaster; wainscoting takes the place of plaster. The ceiling is of wood. At 12 cents a square yard what will it cost to paint the ceiling and wainscoting?

2. At 25 cents a square yard what will it cost to plaster the walls? Take out door and window space.

3. What will the slate blackboard cost at 50 cents a square foot?

4. At 22 cents a bunch what would the laths for the plaster cost?

5. Work the same examples, using the dimensions and arrangements of your own school room.

6. A sitting room 15 x 17 feet and 12 feet high has two doors and three windows, same size as those in example one. It has a one foot base board. At 24 cents a square yard what will it cost to plaster walls and ceiling?

7. What will the laths cost at 27 cents a bunch?

8. Carpet it with Ingrain at 60 cents a yard. Find cost.

NOTE: Carpets usually look best when laid lengthwise. Ingrains are a yard wide. Brussels are usually 27 inches wide. Find out from the width of the room, how many strips of the carpet's width are necessary to cover the floor, remembering when only a fraction of a strip is needed a whole strip of the room's length must be bought.

9. Carpet the room with Brussels at $1.20 a yard. Find cost.

10. Wall paper is $1\frac{1}{2}$ feet wide and is sold in rolls of 24 feet, or usually in double rolls of 48 feet. Paper hangers usually deduct 3 feet in width for each door and window. If a wall is 18 feet long, how many strips will it take to cover it?

11. How many feet of wall paper will it take to cover the walls of a room 12 x 15 and 10 feet high? How many double rolls? How many double rolls if two doors and three windows are taken out?

12. If wall paper costs 20 cents a double roll, what will it cost to paper the room in example six?

13. At that price, what would it cost to paper the school room in example one?

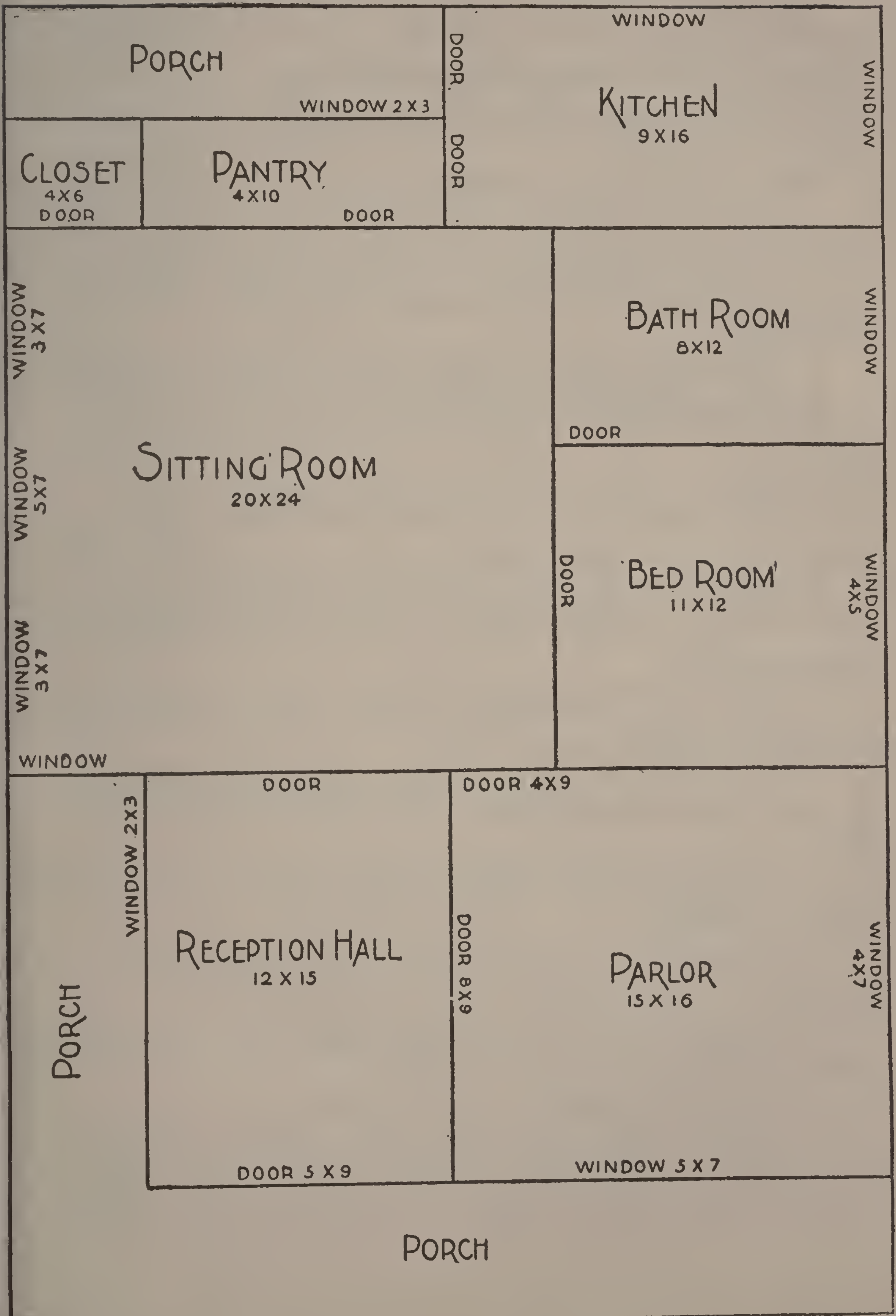
14. At 25 cents a double roll, what will it cost to paper your mother's sitting-room?

15. What will Ingrain carpet cost for mother's sitting-room at 60 cents a yard?

16. What would a Brussels carpet cost at $1.10 a yard?

LESSON IX.

MORE PRACTICAL PROBLEMS.



This is the floor plan of a plain, rectangular house 32 x 48 on a north-west corner lot 40 x 120 feet. It stands 20 feet from the sidewalk in front, and one foot from the north lot line. The rooms are 12 feet high and reach to the eaves. It has a plain roof, reaching over the porches.

Doors are 4 x 9, and windows 4 x 7, unless otherwise indicated.

1. What is the cost of a tight board fence on the north and west side of the lot? Fence 5 feet high; posts 10 feet apart; two 2 x 4 stringers; lumber in the fence costing $20 per M; posts 18 cents apiece.

2. What will it cost to sod the yard at 12 cents a square yard?

3. What will a 6 foot sidewalk of 1½ inch lumber cost on the south and east side? Use three 4 x 4 stringers. Price of lumber $25 per M.

4. What would the same sidewalk cost when made of cement costing 90 cents a square yard?

5. Cost of a 3 foot sidewalk from the front walk to the back of the house. Use inch lumber and two 2 x 4 stringers. Lumber, $24 per M.

6. Cost of 10 x 10 timbers for sills, and one beam running lengthwise, at $27.

7. Cost of 2 x 10 joists, running from center beam to sides, two feet apart, costing $21. What length of joist would you buy?

8. Cost of the 4 x 4's used on every corner of outside and partitions, at $20.00.

9. Cost of the 2 x 6's used as rafters, set two feet apart, each 22 feet long. Price, $18.00.

10. Cost of the roof boards, extending a foot over each gable end. Price, $16.00.

11. Cost of the shingles at $3.50 per M.

12. Find the cost of the floor in the porches at $25.00.

13. Hard wood flooring costs $32 per M. It is 3 inches wide, but on account of the matching it lays only 2½ inches. What will

the floor in the reception hall cost? In the sitting room? Bath room?

14. At 18 cents a square yard what will it cost to plaster the sitting-room, deducting doors, windows and a base board one foot wide?

15. What will it cost to plaster the parlor?

16. At 20 cents a double roll find what it will cost to paper the bedroom. The sitting-room. (Include ceiling in both.)

17. There is a 5 foot wainscoting in the bath room and the window is 4 feet above the floor. Cost of wainscoting at $28 per M. What will it cost to plaster the bath room?

18. What will a Brussels carpet, costing $1.50 a yard, cost for the parlor?

19. An Ingrain at 50 cents a yard for the bedroom will cost how much?

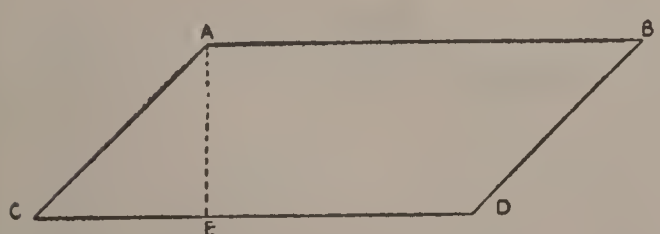
20. At 12 cents a square yard, what will it cost to paint reception hall, bathroom and kitchen, deducting door and window openings?

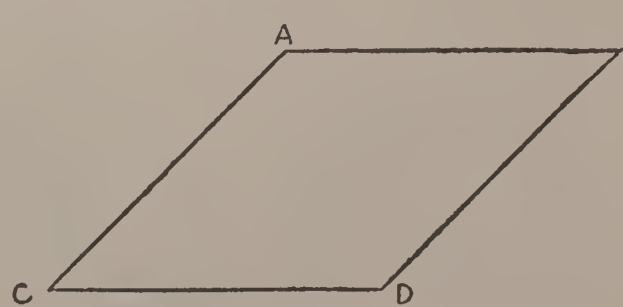
21. At 22 cents a bunch, what will the laths for the closet and pantry cost?

22. At prices given in these problems, construct other problems from the accompanying diagram.

LESSON X.

RHOMBOIDS AND TRIANGLES.





This figure, *a b c d* is called a *rhomboid*. *a b* = *c d* and they are parallel. *a e* is the altitude, it being the perpendicular distance between the two parallel sides of the rhomboid.

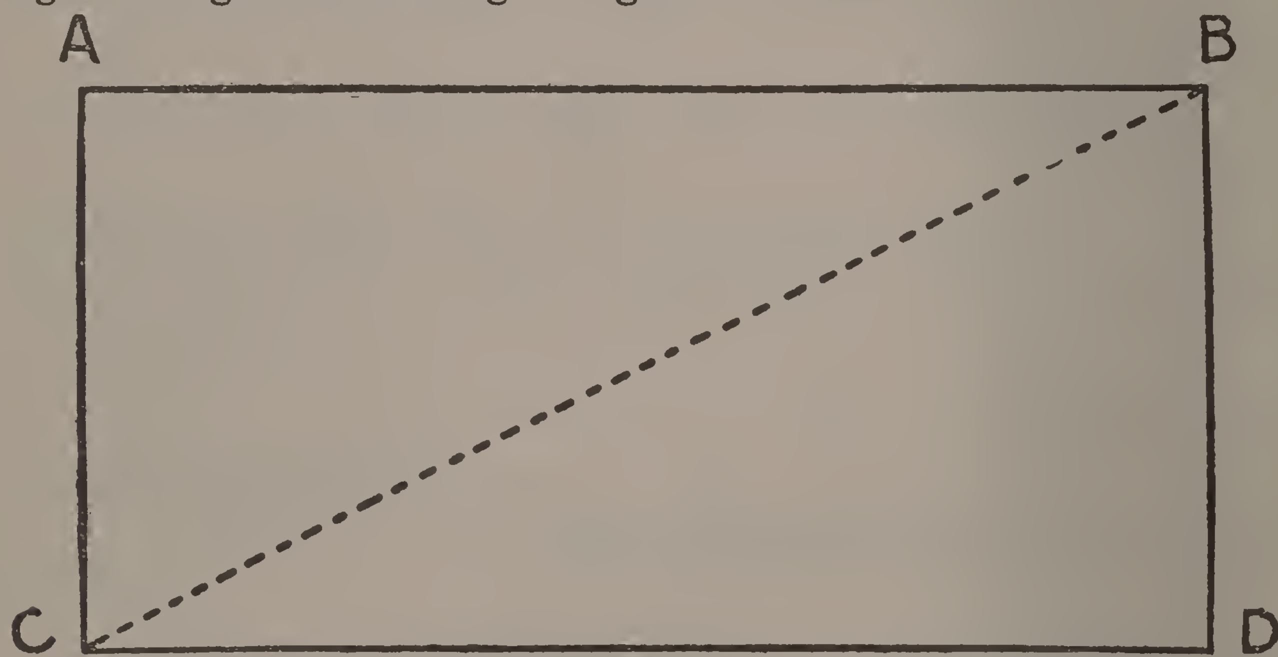
1. Cut a similar rhomboid from card board, and cut off the triangle *a c e*. Fit this triangle on the side *b d*. What kind of a figure have you now? What is its area? What is the area of your rhomboid? How, then, do you find the area of a rhomboid?

2. If the base of a rhomboid is 12, and the altitude 7, what is its area?

3. If the altitude is 16 inches, and the parallel sides 24 inches, what is the area? Parallel sides 18 feet, and altitude 10 inches?

4. Cut a rhomboid 5 inches long and 3 inches wide from a piece of paper. Prove that its area is 15 square inches.

A *Triangle* is a figure having three angles and three sides. A *right* triangle has one right angle.



5. If *a b* is 6, and *a c* is 3, what is the area of this rectangle?

6. What is the area of half of it?

7. Can you see that the dotted line *c b* divides this rectangle into two equal parts, and that the triangle *c a b*, or the triangle *c d b*, is one-half of the rectangle *a b c d*?

8. What is the area of the triangle *c a b*? Triangle *c b d*?

9. In the upper triangle *a b* is the base, and *a c* or *b d* the altitude. In the lower triangle *c d* is the base and *b d* or *a c* the altitude. In the lower triangle *c d* is the base, and *b d* or *a c* the alti-

10. What is its area?

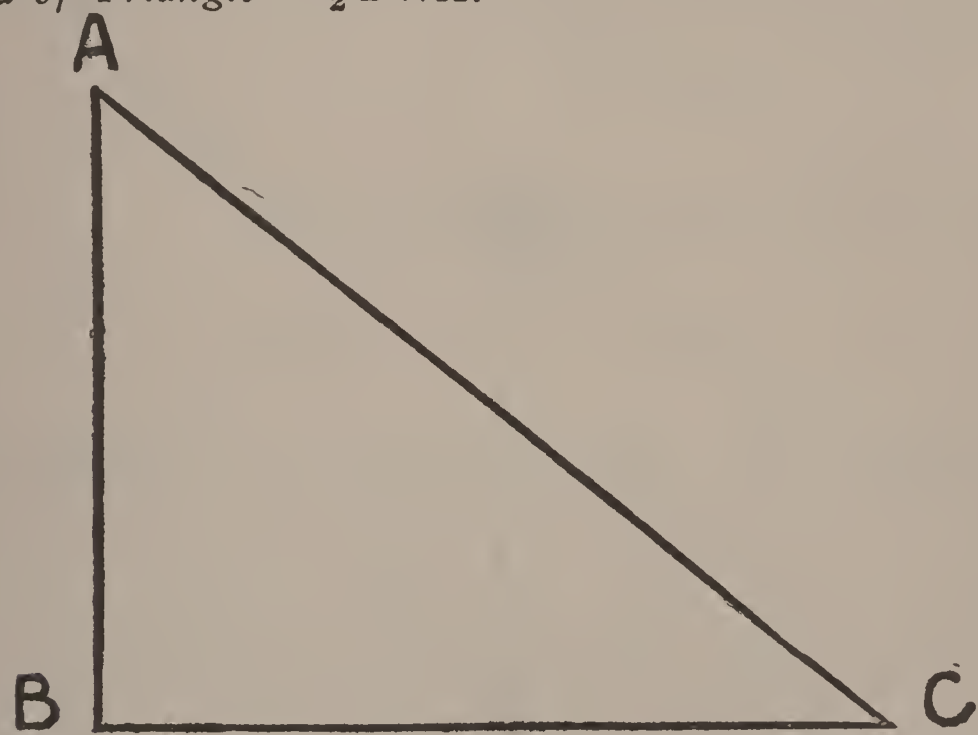
11. What is the base and altitude of the upper triangle?

12. If this upper triangle is one-half of the rectangle, what is its area?

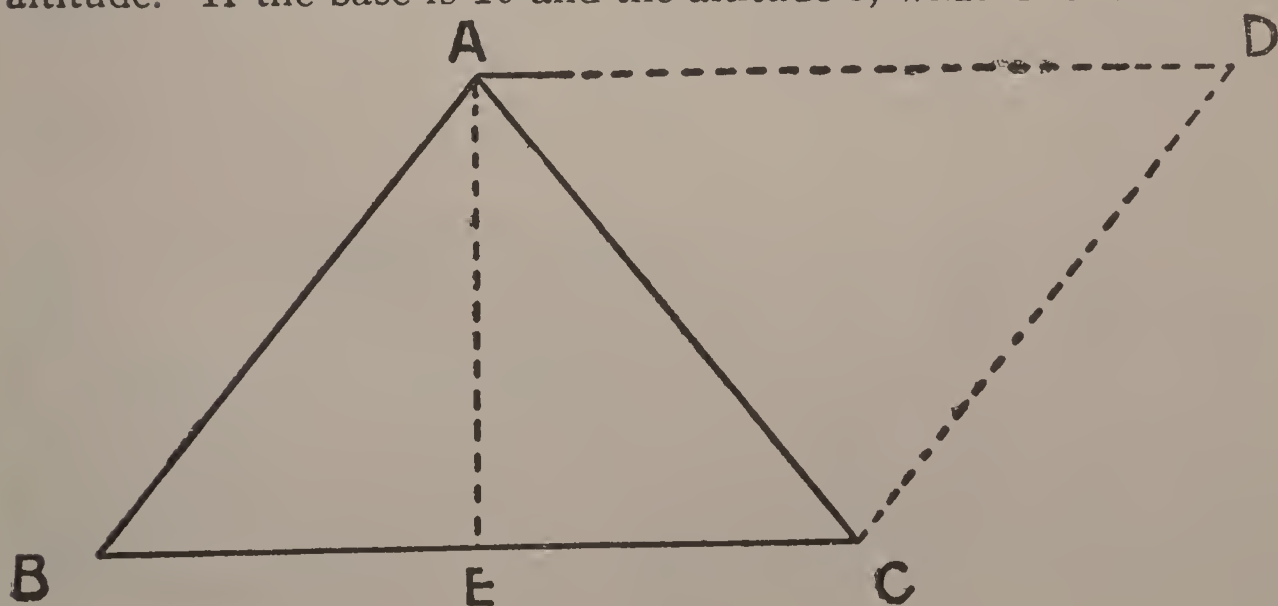
13. If the area of a rectangle equals the base times the altitude (B x A), and a triangle is one-half of a rectangle of the same dimensions, how, then, is the area of a triangle found?

Area of Rectangle $= B \times A$.

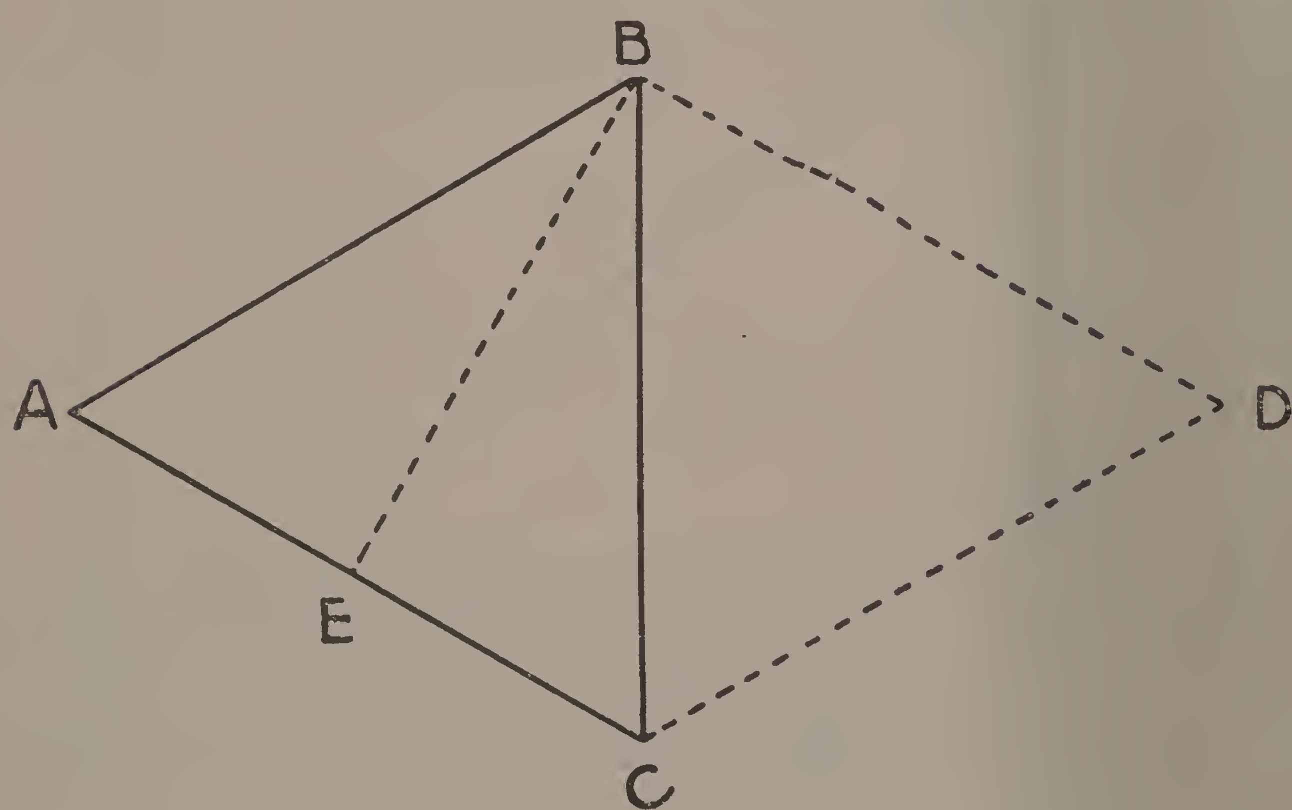
Area of Triangle $= \frac{1}{2} B \times A$.



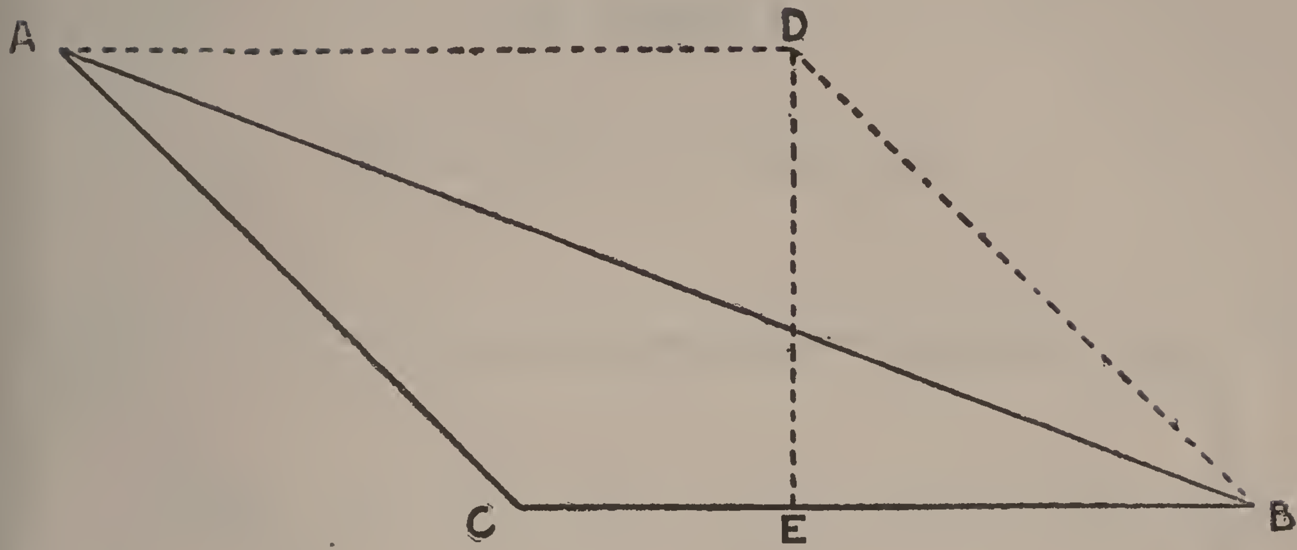
14. This is a *right triangle.* Name its base by letters. Its altitude. If the base is 10 and the altitude 8, what is its area?



15. The triangle *a b c* is an *isosceles* triangle, the side *a b*= the side *a c*. Name the altitude of this triangle by letters. Its base. If the base is 8, and the altitude 5, what is the area of the triangle *a b c*? The addition of the dotted lines *a d* and *d c* makes this into what kind of a figure? Is *a c d* an isosceles triangle also? What are its dimensions? What is its area as compared with the area of *a b c*?



16. The triangle *a b c* is an *equilateral* triangle, the three sides being equal. Name its altitude. Its base. If its base is 6, and the altitude 5, what is the area of the triangle *a b c*? The addition of the dotted lines *b d* and *c d* makes this into what kind of a figure? Is *b c d* an equilateral triangle also? What are its dimensions? Its area? Is an equilateral triangle an isosceles triangle also?



17. The triangle *a b c* is a *scalene* triangle. You will have noticed by this time that the altitude of any triangle is the perpendicular distance from the base to the top, or *apex*. Name the altitude of the triangle *a b c*. The base. If the base is 16, and the altitude 10, what is the area of *a b c*? What is the area of *a b d*? Can you see that every triangle can be extended into either a rectangle, as in the case of a right triangle, or a rhombus, as in the case of the equilateral triangle, or into rhomboids as in the case of the isosceles or scalene triangles? What can you say of the dimensions of each triangle and its corresponding four-sided figure?

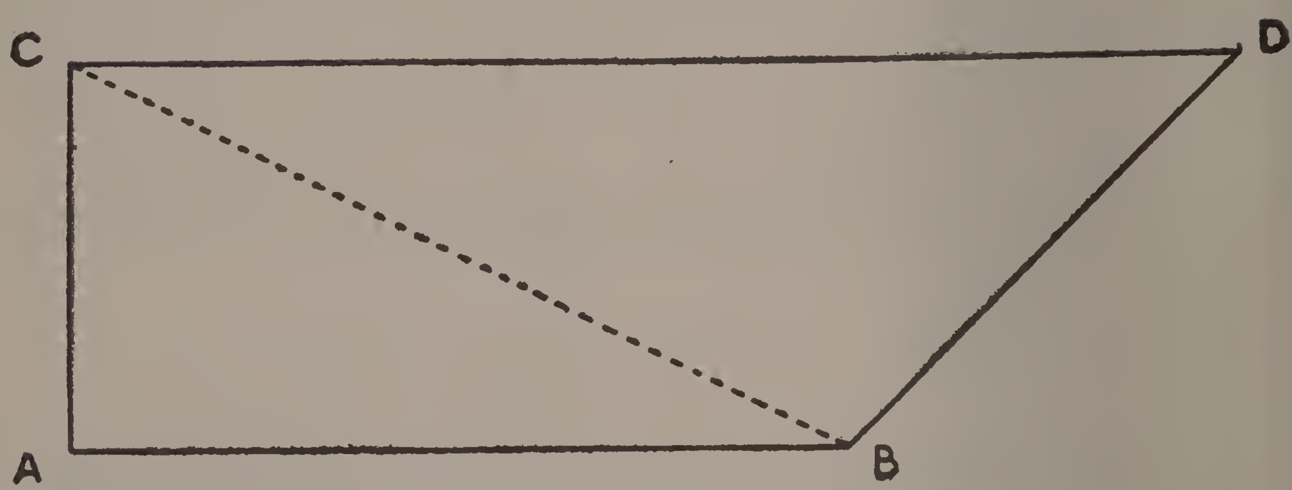
18. The base of a triangle is 8 feet, the altitude is 12 feet; find its area.

19. Stake off a large triangle on the school ground, and find the area of it by measuring its base and altitude.

20. If a barn is 40 feet wide and the perpendicular distance between the eaves and the ridge pole is 20 feet, how much inch lumber is necessary to cover the gable end? To cover both ends?

LESSON XI.

TRAPEZOIDS AND POLYGONS.



1. A necessary condition in a *trapezoid* is that the two bases are parallel. In the above trapezoid *a b* is one base and *c d* another. Can you see that the dotted line *c b* divides the above trapezoid into two triangles? Name them.

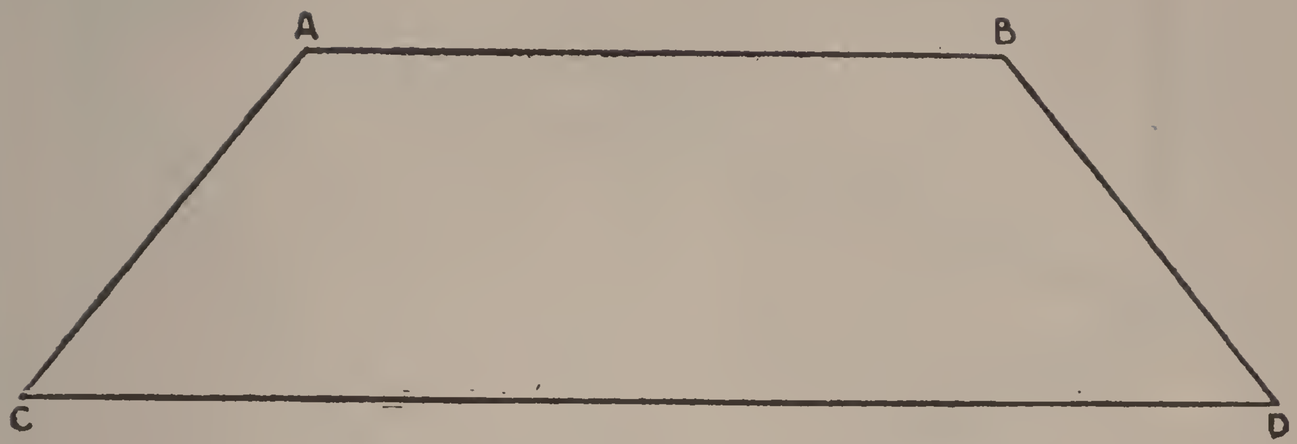
2. What dimensions of the triangle *c a b* do you wish to know in order that you may find its area?

3. If *a b* is 8, and *a c* 4, find the area of that triangle.

4. If the bases are parallel, what is the altitude of the triangle *c b d*? If *c d* is 12, what is the area of *c b d*?

5. What is the area of the whole trapezoid?

6. Draw a trapezoid on the board with one base of 20 inches, the other base of 30 inches, and the altitude 10 inches. Divide it into triangles and find its area.



7. Draw a dotted line from c to b. What two triangles have you?

8. If $c\ d$ is 18, $a\ b$ 12, and the altitude 6, find the area of $a\ b\ c\ d$.

9. Find the area of a trapezoid with one base of 300 feet, another base of 640 feet, and an altitude of 120 feet.

10. Draw a large trapezoid on a piece of paper. Find its dimensions, and its area.

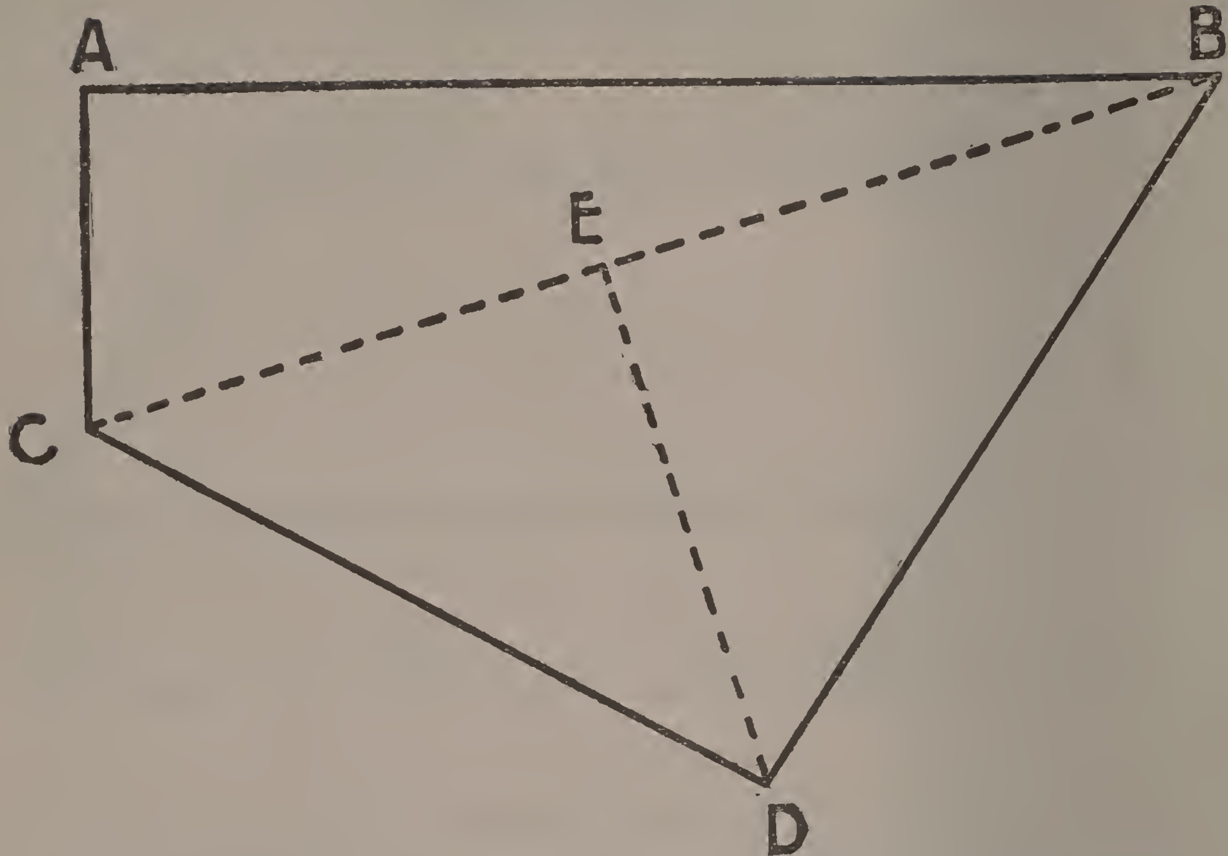
11. Draw another trapezoid on the blackboard. Measure it and find its area.

12. Stake off a piece of ground in the shape of a trapezoid and find its area.

In finding the area of a trapezoid the quickest way is to multiply the sum of the bases by one-half the altitude.

Area of Trapezoid = Sum of Bases $\times \frac{1}{2}$ *A.*

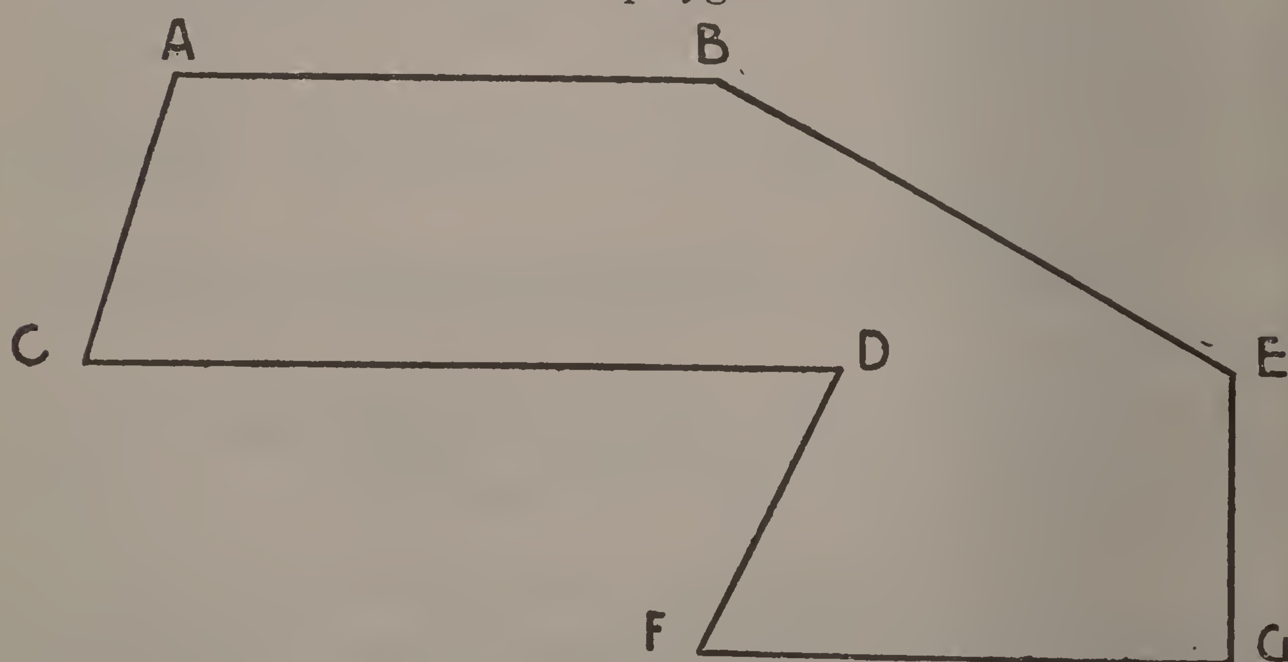
We can find the area of any irregular figure by dividing it into triangles, and finding the area of the triangles of which it is composed.



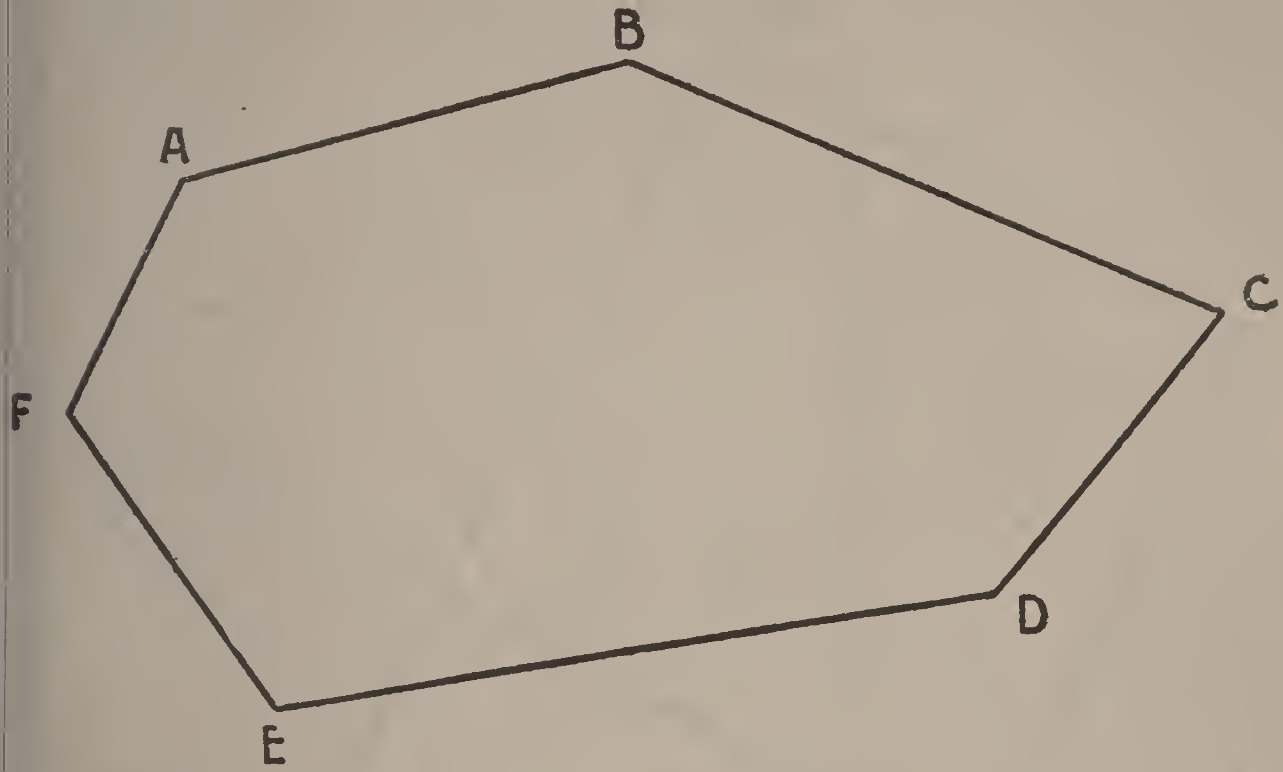
13. The problem is to find the area of the irregular polygon *a b c d*. If *c b* is 8 and *d e* 6, what is the area of the triangle *c b d*?

14. If *a b* is 7, and *a c* 3, what is the area of the triangle *a b c*?

15. What is the area of the polygon *a b d c*?



16. Find the area of the above polygon. *a b* = 8; *c d* = 10; from *d* to *e* = 5; *f g* = 6; perpendicular distance between *a b* and *c d* = 4; between *d e* and *f g* = 5.



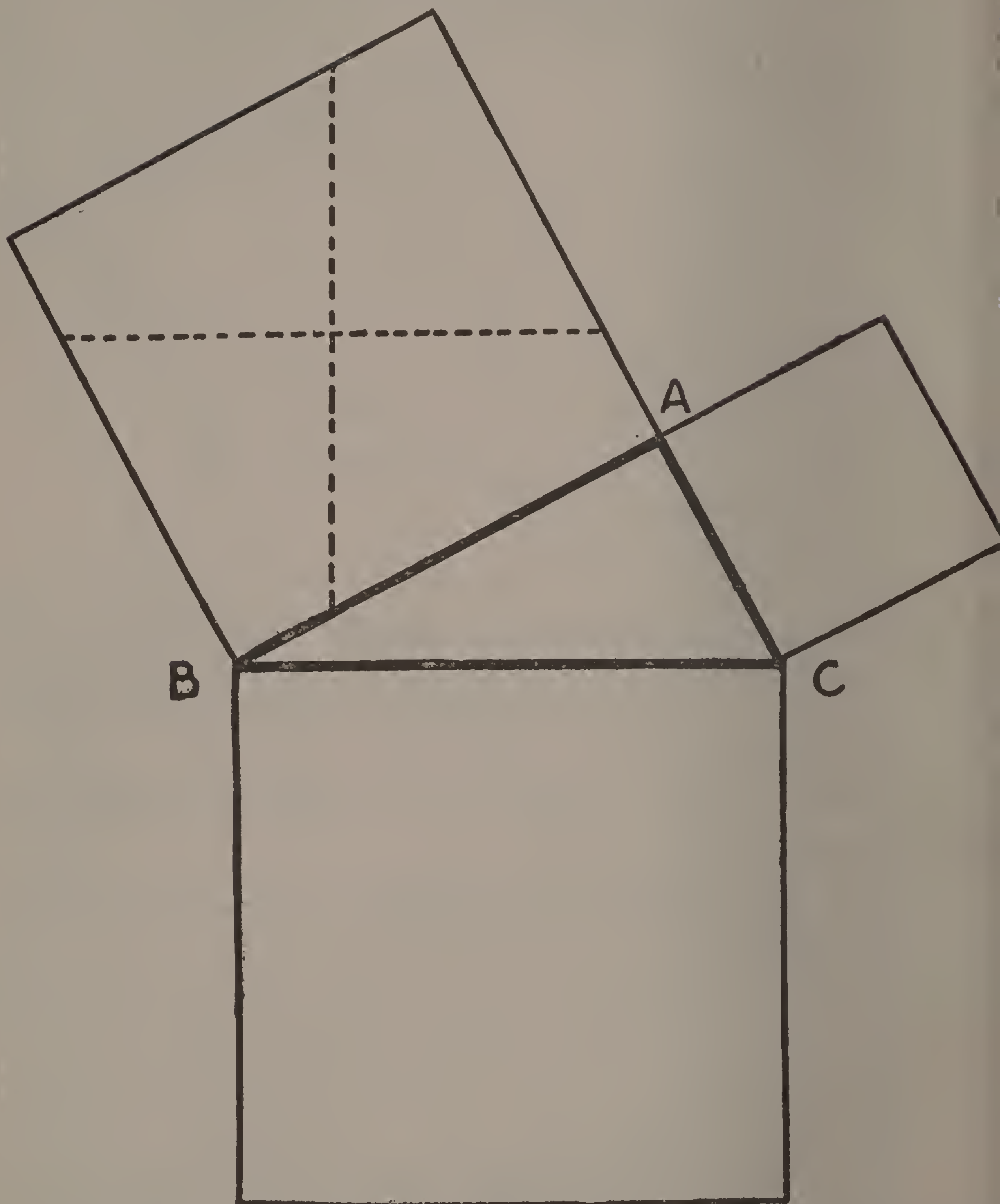
17. Find the area of the above polygon. From f to $b = 16$; f to $c = 20$; e to $c = 18$; the perpendicular distance from the line fb to $a = 5$; fc to $b = 10$; fc to $e = 8$; ec to $d = 4$.

18. Draw a similar polygon on the blackboard and find its area by measurements.

19. Stake off an irregular piece of ground on the school ground and find its area.

LESSON XII.

THE HYPOTENUSE.



The heavy lines *a b c* form a right triangle, with the right angle at *a*. *a b* is the base; what is *a c*? The line *b c* of any right triangle is called the *hypotenuse*. Draw a 2 x 3 inch right triangle on a piece of cardboard. Carefully construct a square on each

side, as is done in the figure above. Determine the center of the square on $a\,b$ by drawing its diagonals. Through this center draw two lines at right angles to each other, and parallel to the sides of the square constructed upon the hypotenuse, $b\ c$, as is shown by the dotted lines in the above figure. This divides the square on $a\ b$ into four parts. Cut out these parts, and also the square on $a\ c$, and place the five pieces so as to cover the square on the hypotenuse. (Place the small square diagonally in the middle, and the other four pieces will fit exactly around it.) What do you conclude?

1. Try this again with a right triangle whose base is 4 inches and altitude 3 inches. It works every time. How many square inches in the square constructed on the base? In the square constructed on the altitude? In both together? How many square inches must there be, then, in the square constructed on the hypotenuse? If there are 25 square inches in the square on the hypotenuse, what is the length of the hypotenuse of a right triangle whose dimensions are 3 x 4 inches? How did you get it?

2. Try the same with a triangle whose base is 8 and altitude is 6.

The square of the base + the square of the altitude = the square of the hypotenuse.

$$B^2 + A^2 = H^2, \text{ or}$$
$$\mathrm{H} = \sqrt{\mathrm{B}^2 + \mathrm{A}^2}, \text{ or}$$

The hypotenuse = the square root of the square of the base + the square of the altitude.

The square of 8 = 64
The square of 6 = 36
100

The square root of 100 = 10, the length of the hypotenuse.

3. Base is 16, altitude 12, find the hypotenuse.

4. Base is 80, altitude 60, find the hypotenuse.

5. Base = 172, altitude = 129, what is the hypotenuse?

6. If 9 + 16 = 25, 9 = 25—what? If $a + b = c$ what is a?

7. If the square of 3 + the square of 4 = the square of 5, what does the square of 4 equal?

8. If the square of the base + the square of the altitude = the square of the hypotenuse, what does the square of the base equal?

9. When you have the square of the hypotenuse given, and the square of the base, what would the square of the altitude equal?

10. If the square of the hypotenuse is 25, and the square of the base is 16, what is the square of the altitude? What is the altitude?

11. If the base is 4, and the hypotenuse 5, what is the altitude? How did you get it?

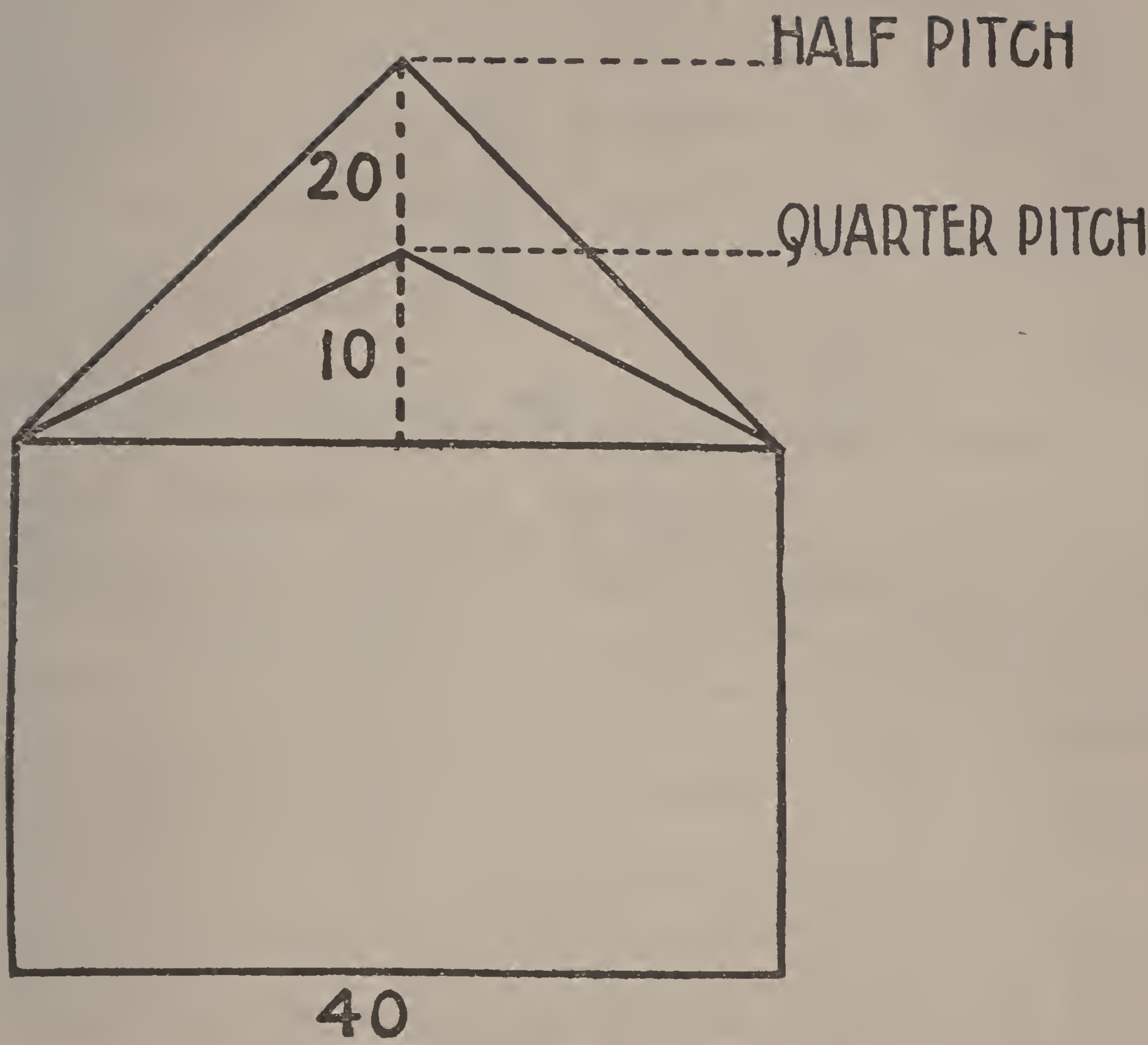
12. You have learned that the square of the hypotenuse, minus the square of one side, equals the square of the other side, hence, the other side equals the square root of this difference. Hypotenuse 10, altitude 6, find base.

13. Find the side indicated by a dash:

BASE.	ALTITUDE.	HYPOT.	BASE.	ALTITUDE.	HYPOT.
a. —	30	34	k. —	60	75
b. 10	24	—	l. 40	—	50
c. 7	—	25	m. ¾	⅔	—
d. —	80	82	n. 18	24	—
e. 20	—	29	o. —	48	52
f. 12	16	—	p. 21	72	—
g. —	40	41	q. 36	—	60
h. 9	—	15	r. 90	—	150
i. —	15	17	s. 5	—	13
j. 14	—	50	t. 36	105	—

14. A tree broke off, and the top fell 36 feet from the stump; the part that broke off was 60 feet long, how long was the stump?

15. How far is it from one corner to the opposite corner of your school room?



16. If the top, or ridge pole, of a roof is one fourth of the width of a building above the horizontal line connecting the two eaves, the roof is said to be a *quarter pitch*; if one half the width of the building, a *half pitch*. From this we may determine the length of the rafters. What is the length of the rafters for a quarter pitched roof in a 40 foot wide building?

17. For a half pitched roof?

18. A building is 20 feet wide; what is the length of the rafters in a half pitched roof?

NOTE: A practical carpenter, however, will not stop to figure the length of rafters. He will nail together a right triangle made of lumber, and measure the length of its hypotenuse, which is the length of the rafters.

LESSON XIII.

PRACTICAL PROBLEMS.

1. The base of a rectangle is 4, the altitude is 3, what is the area?

2. The area of a rectangle is 12, the base is 4, what is the altitude? How did you get it?

3. The area of a rectangle is 20, the base is 5, what is the altitude? ($5 \times \text{what} = 20$?)

4. Find the base of a rectangle whose area is 640, and the altitude is 32. Area 84, base 12, find altitude.

5. Draw a 4 x 6 inch rectangle. Draw a diagonal of that rectangle, a line from one corner to the opposite corner. You now have two equal triangles. What is the area of one triangle?

6. The base of that triangle is 6, what is the altitude?

7. The area of a triangle is 20, the base 5, what is the altitude? How did you get it?

8. The area of a triangle is 40, the base is 10, what is the altitude?

9. Area 320, base 64, find altitude.

10. What is the length of a farm 80 rods wide, and containing 75 acres?

11. The area of a field is 60 acres, one side is 60 rods, what will it cost to build a fence around it at $1.20 per rod?

12. The area of a triangular field is 60 square rods, one side is 30 rods, what is the distance around it?

13. The longest side of a right triangle is 50 yards, and the shortest side is 90 feet, what is the length of the third side?

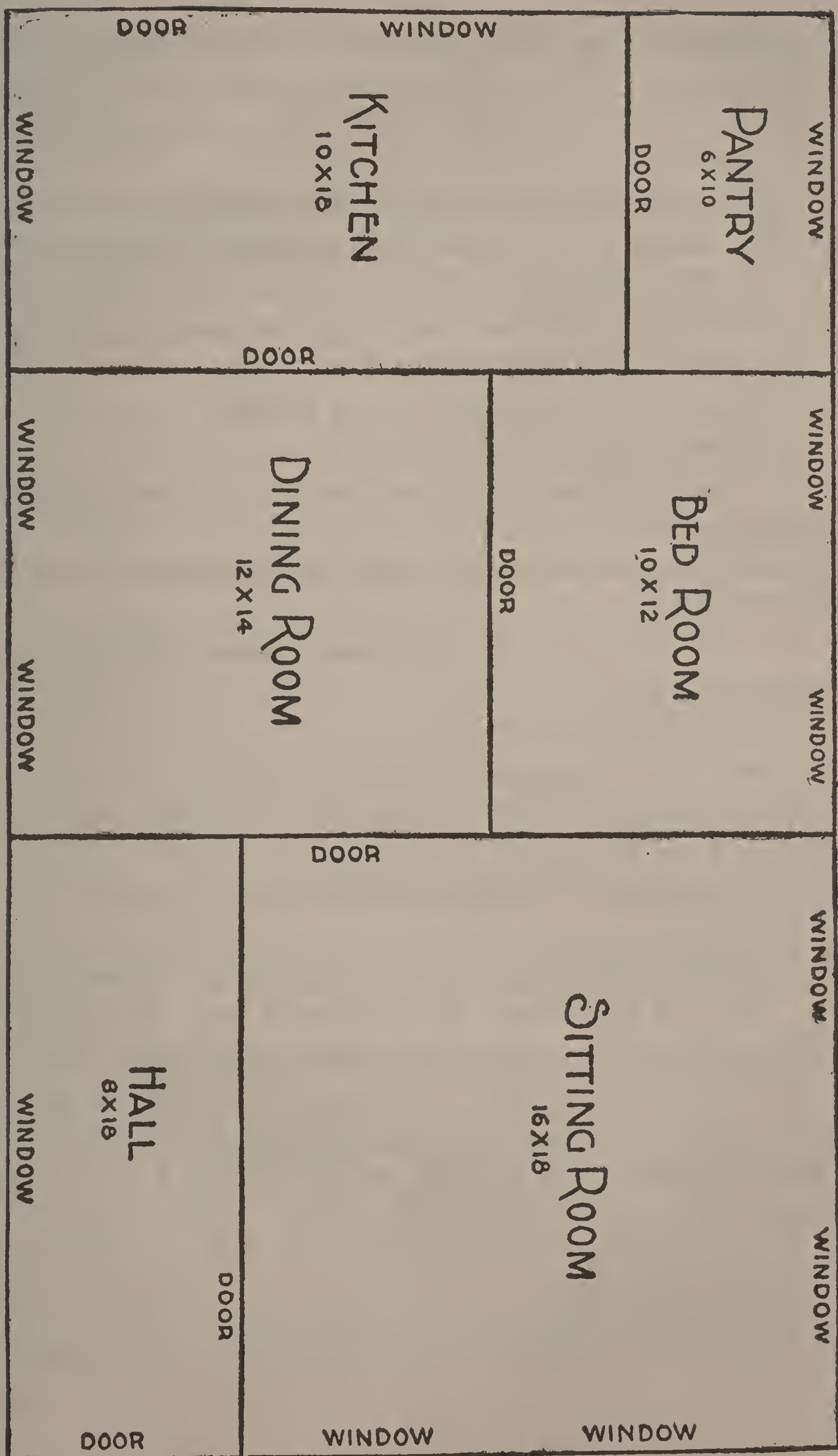
14. What is the distance around a right triangle whose base is 112 feet, and altitude 28 yards?

15. What is the distance around a rectangle whose base is 100, and whose diagonal is 125?

16. If a pole 96 feet high casts a shadow 128 feet long, what is the distance from the top of the pole to the farther end of the shadow?

17. A six acre rectangular field is 660 feet long; how far around it?

18. If the foot of a ladder 50 feet long is put 30 feet from the base, how far up will it reach?


KITCHEN
10X18
PANTRY
6X10
DINING ROOM
12X14
BED ROOM
10X12
HALL
8X18
SITTING ROOM
16X18
DOOR
WINDOW

This diagram is the floor plan of an ordinary frame house, 40 feet long, 24 feet wide and 13 feet high to the eaves. The windows, 3 x 5 feet, and the doors, 3 x 7 feet, are indicated. The house faces the south.

19. How many square feet of ground does the house cover?

20. If it stands in the middle of a square lot containing 2,500 square feet, how much of the lot is not covered by the house?

21. How many square feet in the lot lying north of the house?

22. Lying west of the house?

23. What is the distance around the house?

24. Distance around the lot?

25. How far is it from one corner of the lot diagonally across to the other corner?

26. How far is it from one corner of the house to the same corner of the lot?

27. How far is it from the north-east corner of the house due east to the lot line?

28. Due north to the lot line?

29. How far is it diagonally across the house?

30. From the south-west corner of the house to the north-east corner of the lot?

31. If posts are set 10 feet apart, how many posts are needed around the lot?

32. Put up a tight board fence, 6 feet high, on three sides of the lot. Use 2 x 4 stringers to nail boards on. If the lumber costs $25 per M, what would the lumber of that fence cost?

33. Put up a picket fence in front of the house, and use stringers as above. How many feet of lumber in the stringers? How many pickets, each 4 inches wide, and set 4 inches apart, are needed?

34. What length of boards should you buy in example 32? What length scantlings?

35. At 25 cents a square yard, what will it cost to paint the board fence?

36. Build an 8 foot walk of $1\frac{1}{2}$ inch lumber, costing $25 per M, on the street in front of the house, on three 4 x 4 stringers, costing $20 per M. Find cost.

37. Find cost of a similar sidewalk from the front of the house, flush with the west side of the house, to the street.

38. Build a 4 foot walk on the east and north sides of the house, and on the south side to the front walk. Use inch lumber, and only two stringers. Find the cost of the lumber at $25 per M.

39. The sills under the house are 12 x 12 timbers, how many feet of lumber in them?

40. Three pieces of timber, 8 x 8, are laid across the width of the house, how many feet of lumber in them?

41. Place these three pieces of timber in such a manner that the foundation rectangle is divided into four equal rectangles. Without considering the width of the timbers, what would be the width of each rectangle?

42. Place 2 x 8 joists across the width of these rectangles, each 2 feet apart, and find the number of feet of lumber in the joists.

43. Remember, the house is 12 feet high to the eaves from the top of the sill; place 4 x 4 scantlings at each joint where two walls meet. Find how many feet of lumber in the scantlings.

44. If 2 x 4 scantlings are placed every two feet, what is the cost of the scantlings around the outside wall at $22 per M? (Do not count the 4 x 4's already up. Doors and windows not considered.)

45. In like manner find the cost of the scantlings in the partitions.

46. All the scantlings in the walls and partitions are up. Now plates are necessary to hold them together at the top. Two 2 x 4's are usually nailed one on top of the other. What would they cost at $22 per M?

47. Joists, 2 x 8, are necessary to support the ceiling. They are nailed to each scantling across the building, and are two feet apart. At $24 per M what would they cost?

48. Put on the rafters that are to hold the roof. If they are placed two feet apart, how many does it take for one side of the roof? How many for both sides?

49. The ridge pole, or highest point where the two sides of the roof meet, is 12 feet high from the top of the end scantlings already up; how long must the rafters be if they are extended one foot over the eaves?

50. If the rafters are 2 x 4 scantlings, how many feet of lumber in all of them?

51. How many feet of lumber in the scantlings needed for the gable ends?

52. Now the frame work is up. Cover the four walls with clap-boards at $15 per M. Find how much they will cost. (Doors and windows are not taken into consideration.)

53. What would it cost to cover the gable ends?

54. Find the cost of the boards that cover the roof, extending them one foot over each gable; price, $16 per M.

55. At 90 cents per bunch, find the cost of the shingles to cover the roof.

56. At 20 cents per square yard, what will it cost to paint the roof?

57. Cover the sides with siding at $28 per M, laying it 4 inches to the weather. A 6 inch siding will cover only a 4 inch width.

58. What will it cost to cover the gable ends with siding?

59. At 30 cents a square yard, what will it cost to paint that house?

60. If the doors cost $7.00 apiece, and the windows $7.50, find the cost of the doors and windows.

61. Cover the sitting-room with ordinary flooring 1½ inches thick, at $30 per M. Find cost.

62. How much would it cost to put a floor in the hall?

63. The dining-room?

64. Put a 2 inch hard wood floor in the kitchen at $30 per M. Find cost.

65. How much would the same quality of floor for the pantry cost?

66. Find the number of feet of inch lumber necessary for a base-board one foot high for the sitting-room.

67. A like base-board for the bed room.

68. A $1\frac{1}{2}$ foot base-board for the dining-room.

69. A 3 foot wainscot for the hall.

70. Put a 4 foot wainscot in the kitchen; how much lumber will it take if 10 % is added for matching?

71. At 24 cents a bunch, what would it cost to lath the kitchen? Do not forget the ceiling, and subtract the wainscot.

72. At 18 cents a square yard, what would it cost to plaster the sitting-room?

73. Find what it would cost to lath the kitchen, without the ceiling.

74. The ceiling is covered with fine matched inch lumber costing $28 per M. Add 15 % for matching and find cost.

75. How much lumber in a 4 foot wainscot in the pantry, adding 10 % for matching?

76. At 25 cents per double roll, find cost of papering the sitting-room.

77. A border at 3 cents a yard is put around the top next to ceiling. Find cost.

78. What would it cost to paper the dining-room if the paper costs 20 cents per double roll?

79. Cost of the border at $2\frac{1}{2}$ cents per yard.

80. The sitting-room is carpeted with a carpet $\frac{3}{4}$ yard wide. At $1.25 per yard, find cost.

81. Carpet paper 1 yard wide is placed under the carpet in the sitting-room, and costs 5 cents a yard. Find cost.

82. The hall is covered with oil cloth 32 inches wide, and costs 40 cents a yard. Find cost.

83. Find the cost of the carpet paper in the hall.

84. A carpet 1 yard wide, costing 50 cents a yard, is placed in the bed room. Find cost.

85. What is the diagonal distance across the sitting-room?

86. How far is it from the upper north-west corner to the lower south-east corner of the sitting-room?

87. Suppose a spider travels from the upper north-east corner of the dining-room straight down to the floor, diagonally across to the opposite corner, up to the upper south-west corner and diagonally across to the starting point. What distance has it traveled?

88. What kind of a figure has it described?

89. What is its area?

90. What is the length of its diagonal?

91. What is the diagonal distance across a window?

92. If the dimensions of the sitting-room were doubled, what would its area be?

93. What would be the dimensions of the bed room if it were twice as large?

94. How long must a ladder be to reach to the ceiling and stand 4 feet from the wall?

95. Find how far it is from the top of the north-east corner of the 6 foot board fence to the north-east corner of the eaves.

96. A fly travels from the lower north-east corner of the sitting-room up 10 feet and from there flies to the upper south-west corner, then down to the lower corner and diagonally across to the starting point. How far has it traveled?

97. What kind of a figure has it described?

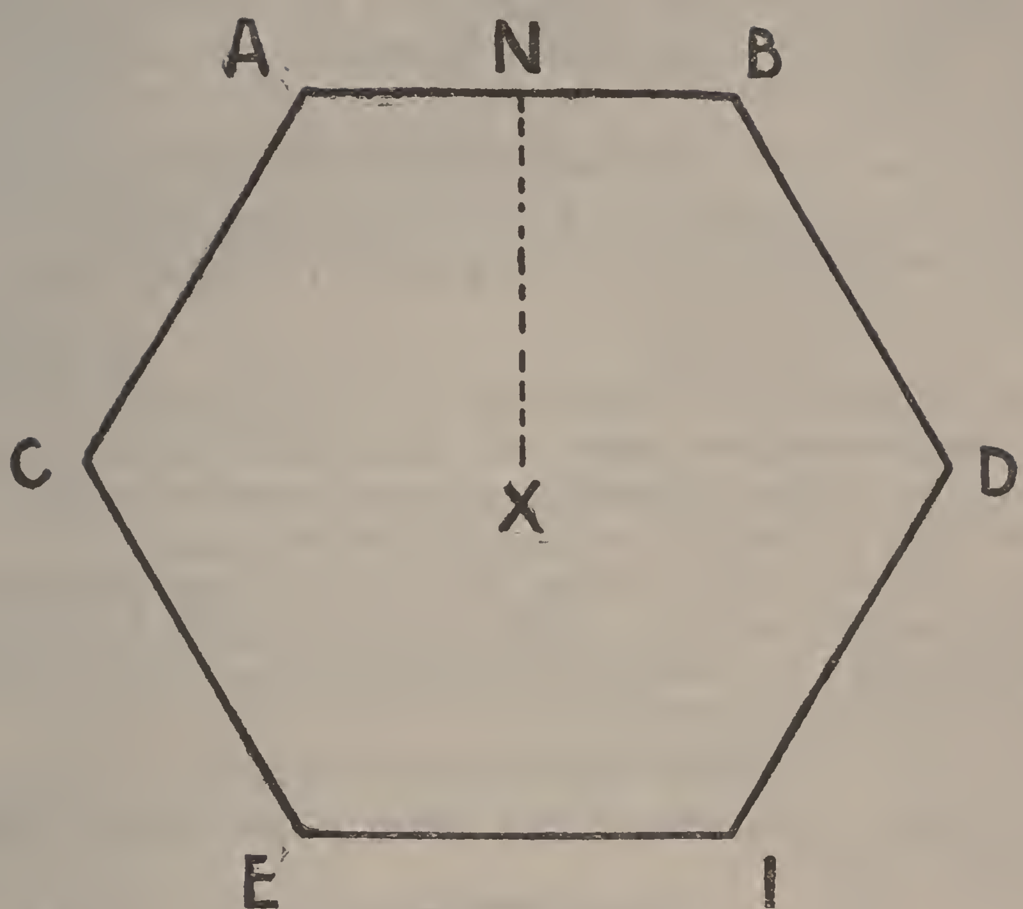
98. Find its area.

99. How much of the north end of the hall must be cut off to make just a square yard?

100. A lightning-rod is put on the house, which stands 5 feet on the ridge pole and is placed 6 feet into the ground. How long is it?

LESSON XIV.

CIRCLES.



1. This is a *hexagon*; *hex* = six, *gon* = angles. The best way to draw one is to inscribe it in a circle, the radius of the circle being exactly the length of one side of the hexagon. Draw one a little larger than this one on a piece of pasteboard.

2. Find the area of this hexagon if the line *n x* is 5 and one side 6. The best way to do this is to draw the diagonals. How many triangles have you now? Are they equal? What is the area of each?

3. Cut the hexagon that you have drawn into six equal triangles and lay them in a row thus:

4. What is the length of *a b*? What is the altitude of each triangle? What is the area of that figure?

5. Can you see that the line $a\ b$ corresponds to $a\ c\ e\ i\ d\ b$, or the distance around the hexagon, and the altitude corresponds to $n\ x$? Then, how may we find the area of any hexagon?

NOTE: The right name for the line $n\ x$ is *apothem*, and for the sum of the lines around it, *perimeter;* but since our purpose is to work towards a circle, we shall call $n\ x$ the *radius* and $a\ c\ e\ i\ d\ b$ the *circumference.*

Area of Hex. $= \frac{1}{2}$ *radius* $\times$ *circumference.*

6. If the circumference is 25 and the radius is 4, what is the area?

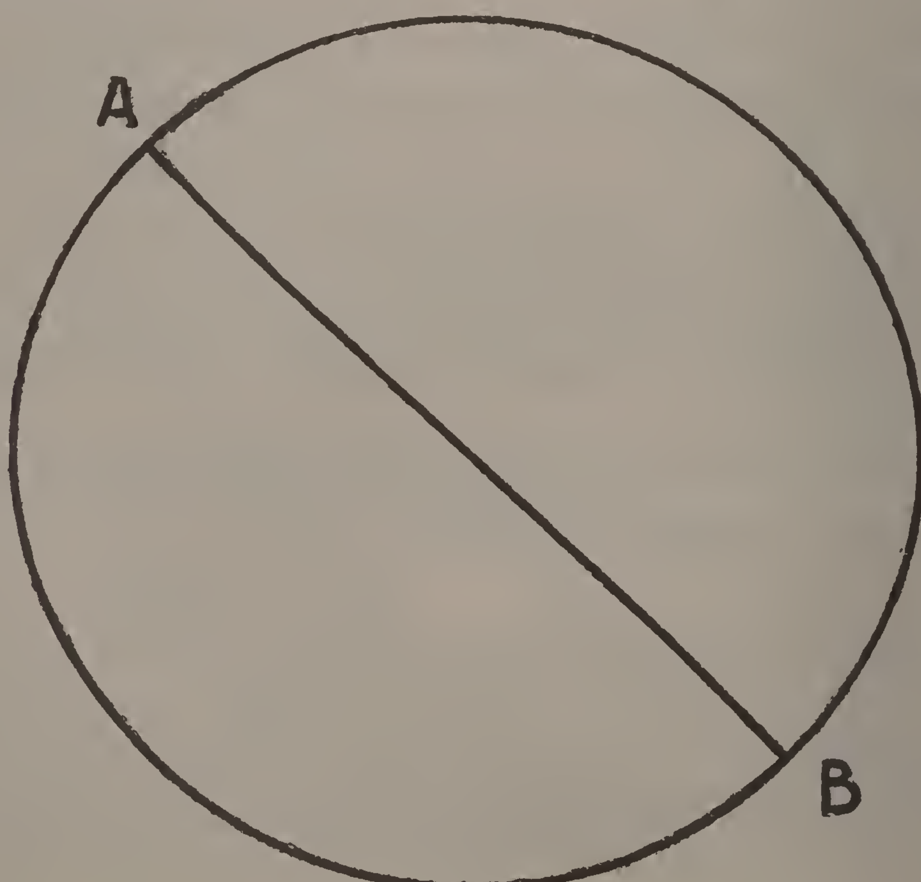
7. Draw a regular figure with 12 equal sides. Can you find its area the same way as you did that of the hexagon?

8. Imagine a figure with 100 equal sides. How would you find the area of that? That figure would be almost a circle. Find its area if the circumference is 50 and the radius 8.

9. A circle is really nothing more than a regular figure with an infinite number of sides. If the area of a regular figure is found by multiplying $\frac{1}{2}$ the radius by the circumference, how would you find the area of a circle?

Area of Circle $= \frac{1}{2} r \times$ *Circum.*

10. If the circumference of a circle is 100 and the radius 16, what is the area?



11. The line $a\,b$ is called the *diameter*. How many radii in a diameter? Diameter $= 2\,r$. Remember that.

12. With a tape measure find the distance around a fruit can. Find the diameter.

13. Measure the circumference of other circular objects and also other diameters. Place the circumferences in one column, and the corresponding diameters opposite each in another column. About how much larger is each circumference than its corresponding diameter?

14. More exactly, the circumference is about 3.1416 times the diameter. This number, 3.1416, is called *pi* and is written π.

(a) *Circumference* $= \pi \times$ *Diameter*, but

(b) *Diameter* $= 2\,r$, hence,

(c) *Circumference* $= \pi \times 2\,r$.

You have learned in example nine that

(d) *Area of Circle* $=$ *Circumference* $\times \frac{1}{2}\,r$.

Now, instead of using the word circumference in equation (d), let us substitute the value of circumference, $\pi \times 2\,r$, as given in equation (c), and we have

(e) *Area of Circle* $= \pi \times 2\,r \times \frac{1}{2}\,r$.

Multiplying, $2 \times \frac{1}{2} = 1, r \times r = r^2$, hence,

(f) *Area of Circle* $= \pi \times r^2$.

That is, the area of a circle is obtained by multiplying the square of the radius by 3.1416. If the radius is 2, what is the area? If it is 5? If it is 10?

15. If the diameter of a circle is 5, what is the circumference?
16. If the radius is 16, what is the circumference?
17. The circumference is 31.416, what is the radius?
18. Circumference 25.1328, find radius.
19. Diameter 36, find radius.
20. Radius 8, find area.
21. Diameter 50, find area.
22. Radius 16, find circumference.
23. Circumference 37.6996, find radius.
24. Find area.
25. Circumference 62.832, find area.
26. Radius 9, find area.

LESSON XV.

PROBLEMS.

1. Study this out and see if you can understand it:

$12 = 3 \times 4$, then $\quad m = a \times c$, then $\quad A = \pi \times r^2$, then
$4 = 12 \div 3$; $\quad c = m \div a$; $\quad r^2 = A \div \pi$.

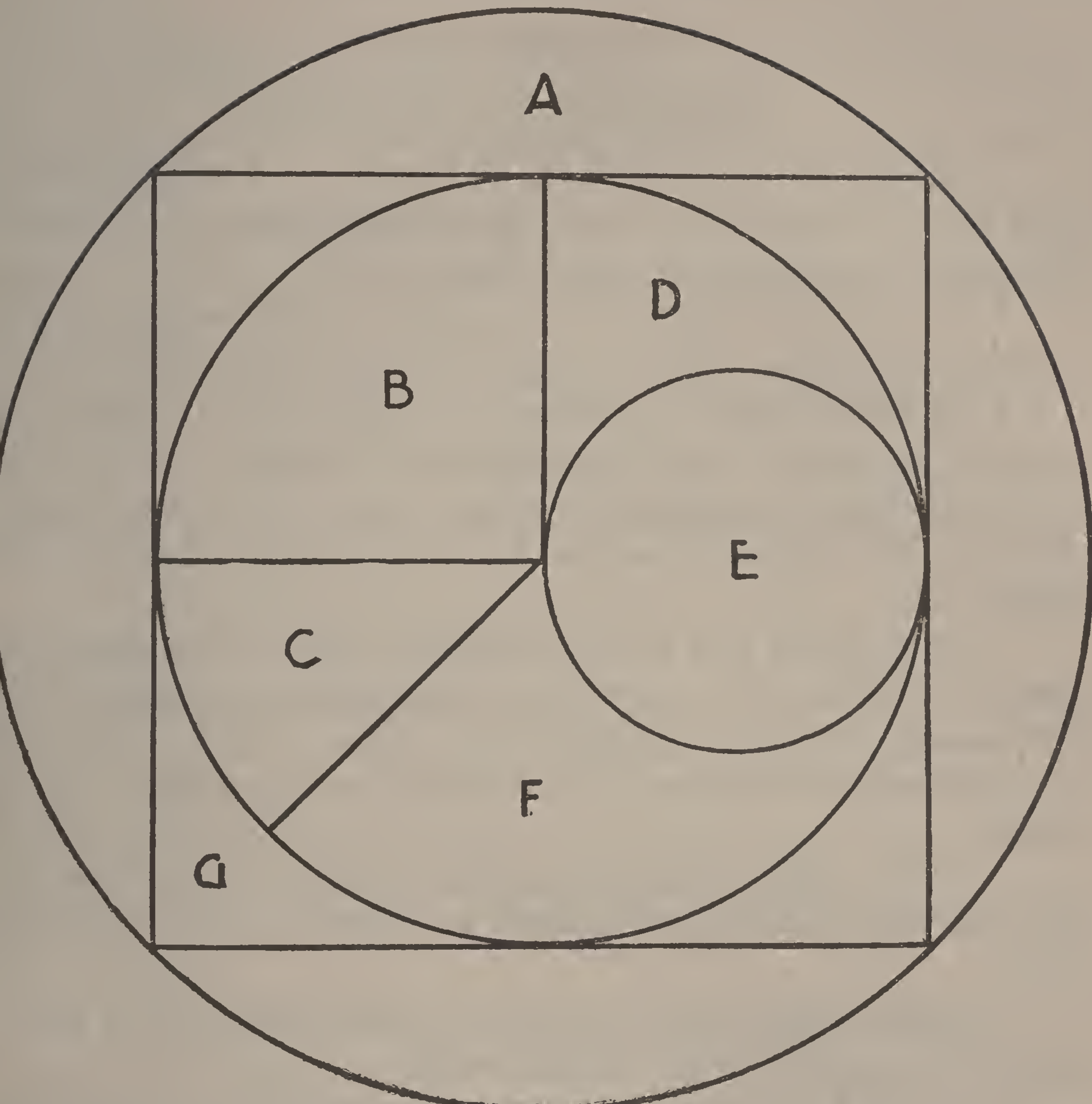
If the square of the radius equals the area $\div \pi$, then the radius equals the square root of the area after it is $\div \pi$.

$r^2 = A \div \pi$, then
$r = \sqrt{(A \div \pi)}$.

27. If A is 12.5664, find r.
$A = 12.5664$; $\pi = 3.1416$;
$12.5664 \div 3.1416 = 4$; $\sqrt{4} = 2$.

Hence the radius is 2 if the area is 12.5664.

2. What is the radius of a circle whose area is 153.9384?
3. What is its diameter? Its circumference?
4. Area is 1320.2543, find diameter.
5. Area is 201.062, find circumference.
6. Find the circumference of a wheel whose diameter is 5 feet.
7. At 90 cents a rod, what will it cost to fence a circular piece of land whose diameter is 25 rods? What is its area?
8. A circular park is 65 rods in diameter, how many acres does it contain? What is its circumference?
9. A horse is attached to a post by a rope 20 feet long. What is the circumference of the circle in which it may graze?
10. What is its area?
11. How many square yards are there in a circle whose diameter is 12 feet?
12. Draw a square containing 81 square inches; inscribe a circle in this square. What is the area of this circle?
13. How much greater or less, at $1.20 a rod, is the cost of fencing a circular piece of ground whose diameter is 12 rods than a square field whose length is 12 rods?
14. One side of the square is 5; what is its area?
15. Find the length of the diagonal of that square. Disregard the fraction.
16. What is the area of the circumscribed, or largest circle?



17. What is its circumference?
18. Area of A? Length of the arc of A?
19. What is the area of the inscribed, or second largest circle?
20. What is its circumference?
21. Area of B? The length of its arc?
22. Area of G?
23. If the angle in C is 45 degrees, what is its area?
24. The length of its arc?
25. Find the area of E. The circumference.
26. Area of D.
27. Area of F. The length of its arc.

LESSON XVI.

MISCELLANEOUS PROBLEMS.

1. Find the length and width of a square box which shall contain as much as 2 boxes, one 2 feet and the other $2\frac{2}{3}$ feet square, the three boxes being the same height.

2. From a lot of land 60 rods square 60 square rods were sold; what was the value of the remainder at $160 per acre?

3. If 2-inch pickets are placed 2 inches apart, how many pickets will be required for a fence half a mile long?

4. 30 feet from a tree 38 feet high, a pole 8 feet long is erected. What is the distance from the top of the pole to the top of the tree?

5. How much less will the fencing of 200 acres cost in the square form than in the form of a rectangle whose length is 5 times its breadth, the price being $2.40 per rod?

6. Find the diameter of a circular island containing 2 square miles.

7. How many persons can stand in a park 20 rods long and 8 rods wide, allowing each person to occupy a space of 3 square feet?

8. How many yards of cotton $1\frac{1}{4}$ yards wide will it take to line 15 yards of cloth $\frac{3}{4}$ of a yard wide?

9. Show that a garden 40 feet square contains the same number of square feet as a garden 20 x 80 feet. At 30 cents a foot, how much more would it cost to fence the second than the first?

10. What is the diameter of the largest circular saw that can be taken through a doorway $8\frac{1}{2}$ feet high and $6\frac{3}{8}$ feet wide?

11. On a level playground there is a rope, $11\frac{1}{4}$ feet long, fastened to a ring at the top of a pole 9 feet high. How far from the foot of the pole will the rope reach to the ground?

12. How many square yards of plastering are there in a room 24 feet long, 18 feet wide and 10 feet high, if the doors and windows take up 150 square feet?

13. A rectangular field contains 20 acres and is 50 rods wide; how long is it?

14. What must be the length of a farm 80 rods wide containing 75 acres?

15. A goat is fastened to the top of a post 40 feet high by a rope 50 feet long. Find the area of the greatest circle over which he can graze.

16. How much larger is a square circumscribing a circle 36 rods in diameter than a square inscribed in the same circle?

17. How much more will it cost, at $\$1\frac{1}{4}$ a rod to fence a field in the form of a rectangle 108 rods long and 48 rods wide, than to fence a field of equal area in the form of a square?

18. The distance around a rectangle is 400 feet. The difference between the length and breadth is 40 feet. Find area.

19. How many square yards are there in the floor, walls and ceiling of a room 20 feet long, 15 feet wide and 10 feet high?

20. A horse is to be tethered in the center of a rectangular lot 240 feet long by 238 feet wide. How long must a rope be which will allow him to graze to the corners of the lot?

21. If the diameter of an iron column is 3.5 inches, find the circumference.

22. Find the area of the bottom of the pail in your school.

23. A man bought a piece of land 1000 feet long and 400 feet wide at 20 cents a square foot; how much did it cost him?

24. In order to make the land available for house lots, he put in two streets, each 50 feet wide, through the center, one in the direction of the length, and the other in the direction of the width. How many square feet did these streets occupy?

25. Illustrate this by diagram.

26. How much did the man make on the investment if, after spending $5000 in the construction of the streets, he sold the remainder at 30 cents a square foot?

27. How many bricks, each $9 \times 4\frac{1}{2}$ inches, will be required to pave a floor $36 \times 27\frac{1}{2}$ feet?

28. A tree 65 feet high casts a shadow 45 feet long. How far is it from the end of the shadow to the top of the tree?

29. The radius of a circle is 5 feet; find the diameter of another circle containing 4 times the area of the first.

30. How many acres in a semi-circular farm, whose radius is 50 rods?

31. Find the area of the lid of the stove in school.

32. A room is 48 feet long, 36 feet wide and 11 feet high. What is the length of a line drawn from one corner of the floor diagonally across to the opposite corner of the ceiling?

33. What is the length of the diagonal of a rectangle, containing $7\frac{1}{2}$ acres, one side of the rectangle being 30 rods?

34. If a man walks $\frac{3}{4}$ of a mile due north, then $1\frac{1}{2}$ miles due west, then in a straight line to the point of starting, around how many acres has he walked?

35. How many square feet are there in a gravel walk 4 feet wide, which runs around the outside of a garden 40 x 28 feet?

36. Illustrate this by diagram.

37. The equatorial diameter of the earth is 7925 miles, how long is the equator?

38. The distance from the center of the hub of a wheel to the outer edge of the felly is 15 inches. How long must the tire be?

39. How wide is a piece of land 80 rods long, which contains 10 acres.

40. How many feet of flooring in your school?

41. Find what it will cost to sod a triangular plot of ground whose base is 100 feet and altitude 33 feet, at 10 cents per square yard.

42. How many acres in a field in the form of a trapezium, divided by a diagonal 130 rods long, into two triangles whose altitudes are 60 and 80 rods respectively?

43. From a lot 40 rods square I sold 40 square rods. Find the value of the remainder at $120 per acre.

44. If the longest side of a right triangular lot is 50 yards and the shortest side is 90 feet, find the length of the third side.

45. If the circumference of a circular pond is 314.16 rods, what part of a mile must I row to pass from shore to shore through the center of the pond?

46. If a horse is tethered to the middle post of a fence, from which he can graze out into the field in a curved line 78.539314 feet long, how long is the tether?

47. The area of a rectangle equals 2704 square feet. What is the length of a square of equal area?

48. Find the number of feet of lumber in a plank 16 feet long, 14 inches wide and $2\frac{1}{2}$ inches thick.

49. Find the value of $1\frac{1}{2}$ inch plank, at $24 per M, that will be needed to make a land-roller whose circumference is 15 feet and length 12 feet.

50. What is the area of both ends of this roller?

51. Find the distance around a right triangle whose altitude is 28 yards and base 112 feet.

52. What will be the circumference of the largest circle that can be drawn on a sheet of paper 12 x 18 inches?

53. What is the area of your school yard?

54. How many fence posts are needed around it, if 12 feet apart?

55. How many feet of fence boards, if built 5 feet high?

56. Build a sidewalk 4 feet wide close around your school house. How many feet of $1\frac{1}{2}$ inch lumber will it take?

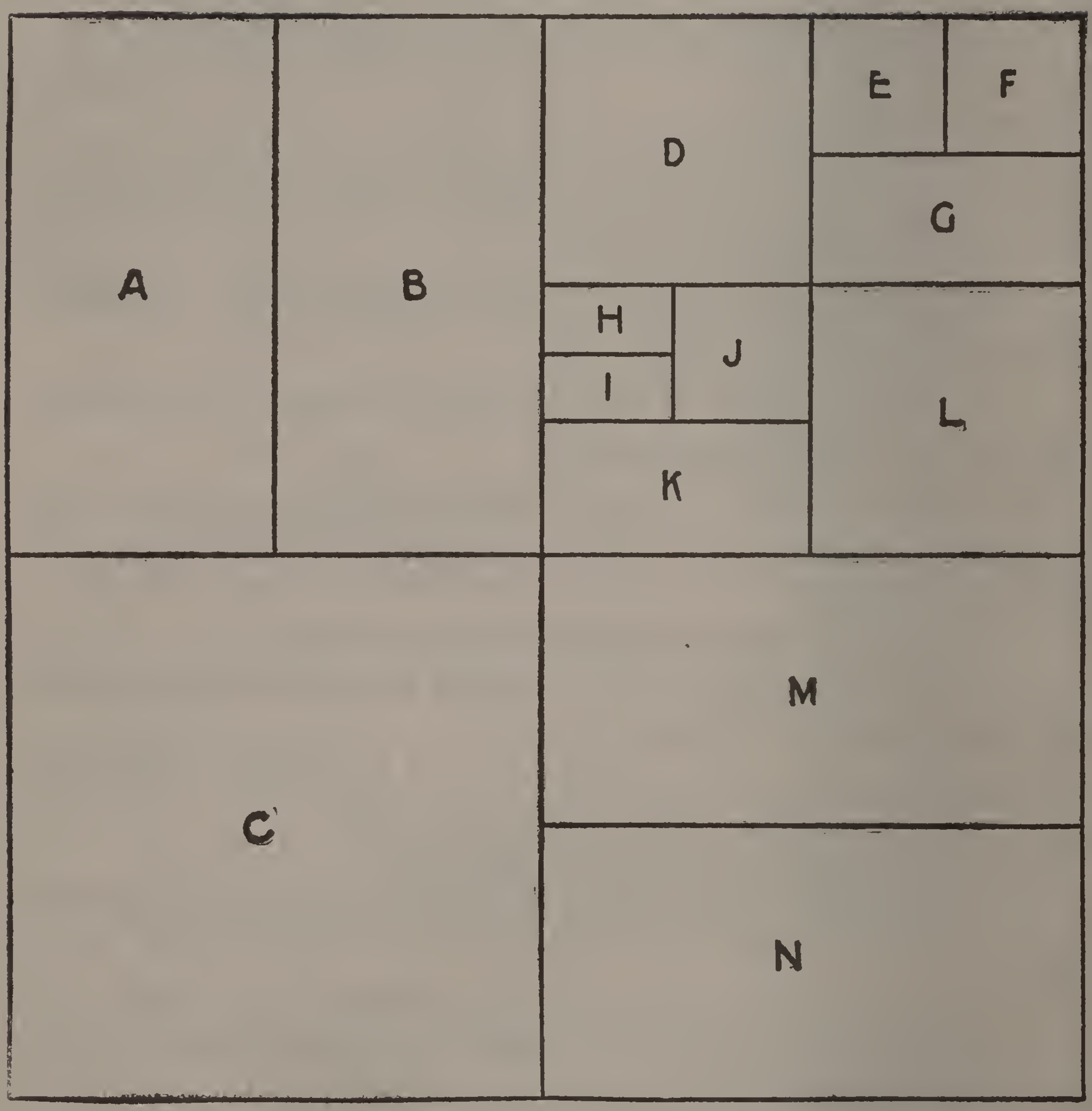
57. How many shingles in the roof of your school?

58. A square yard is cut from a board 18 feet long and 10 inches wide. Find the length of the remainder.

59. How many 8 inch furrows must be plowed across a square 10 acre field to make an acre?

LESSON XVII.

LAND MEASURE.



1. This represents a section of land. How many acres?
2. How many acres in *c*, the S. W. $\frac{1}{4}$?
3. What quarter of the section is $a + b$? (N. W. $\frac{1}{4}$.)
4. What half of the N. W. $\frac{1}{4}$ is b? (E. $\frac{1}{2}$ of N. W. $\frac{1}{4}$.)
5. What half of the N. W. $\frac{1}{4}$ is a?

6. How many acres in *a*?
7. Describe *m*. (N. ½ of S. E. ¼.) Describe *n*.
8. How many acres in *m*? In *n*?
9. What quarter of the N. E. ¼ is *d*? (N. W. ¼ of N. E. ¼.)
10. How many acres in it?
11. Describe *l*. How many acres in it?
12. What half of the N. E. ¼ of N. E. ¼ is *g*? (S. ½ of N. E. ¼ of N. E. ¼.)
13. Describe *e*. Describe *f*. How many acres in each?
14. Describe *k*. How many acres?
15. What ¼ of the S. W. ¼ of the N. E. ¼ is *j*? How many acres?
16. *h* is the N. ½ of N. W. ¼ of S. W. ¼ of N. E. ¼. How many acres?
17. Describe *i*. How many acres in it?
18. How many acres in *h i j*? In *g l d*?
19. How many acres in *b d g*? In *m k l*?
20. In *b i j*? In *c b d*? In *a c n*? In *f g l*?
21. How many acres in E. ½ of N. W. ¼? In S. E. ¼ of N. E. ¼?
22. In N. E. ¼ of N. E. ¼ of N. E. ¼?
23. In N. ½ of S. E. ¼ of N. W. ¼ of N. E. ¼?
24. How many times *k* is *c*?
25. How many times *h* is *m*?
26. How many rods from one corner of the section to the next corner? i. e. How many rods in a mile?
27. How many rods on the eastern boundary of *g*? On the southern boundary of *k*?
28. In the following problems give all distances in rods and measure on boundary lines only. What is the distance from N. E. corner of *c* to S. W. corner?
29. From N. E. corner of *e* to S. E. of *g*?
30. From N. E. of *j* to S. E. of *c*?
31. From N. E. of *h* to N. E. of *n*?
32. From N. W. of *b* to S. E. of *i*?

33. What is the distance around *i*? Around *e*? Around *k*?

34. The distance around *l*? Around *a*? Around *c*?

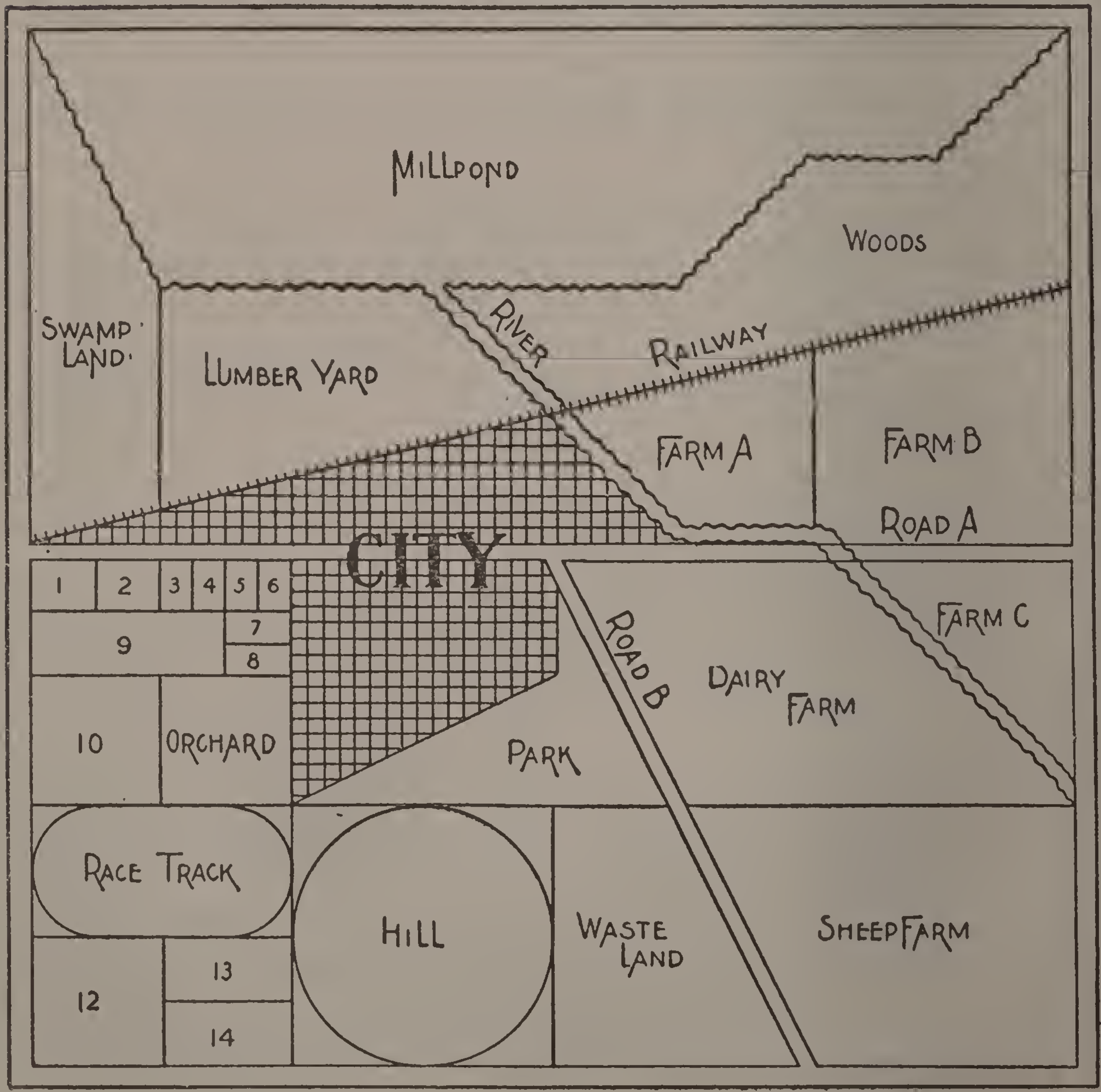
35. Which farm would you rather have, *d h j* or *i j g*?

36. How far is it in a straight line from the N. E. corner of *g* to the S. W. corner of *k*? How far is it by traveling on boundary lines?

37. What part of the section is *a*? *k*? *l*? *e*? *i*?

LESSON XVIII.

MISCELLANEOUS PROBLEMS.



1. This diagram represents four sections of land. With a ruler and pencil carefully divide it into 40 acre squares.

2. Find areas in acres and distances in rods. What is the area of 12? 14? 9? 2? 6? 8?

3. Without considering the width of roads, railway or river in the following series of problems, determine the area of the sheep farm.

4. The area of farm C.

5. Farm B.

6. Waste land.

7. Area of dairy farm.

8. Park.

9. Millpond.

10. Lumber yard. (Notice that the railway cuts across a row of four forties, starting at the S. W. corner of the first forty, crossing the E. boundary of that 40 a $\frac{1}{4}$ way up from its southern boundary, crossing the E. boundary of the second 40 a $\frac{1}{2}$ way up, etc.)

11. Area of swamp land.

12. Of farm A.

13. Of the woods. City (on both sides of the road).

14. The distance around the hill.

15. Area of the hill.

16. Area of the quarter section not occupied by the hill.

17. If the river is 10 rods wide, how much land is taken up by it?

18. How much land is taken up by the railway, 8 rods wide? By road B, 4 rods wide?

19. What is the distance around the city limits?

20. The proprietor of the orchard had to build a 12 foot high fence on the city side. Find the cost of this fence if lumber is worth $25 per M, and posts cost 60 cents apiece. The posts are set 10 feet apart, and three 2 x 4 stringers are used.

21. The summit of the hill is 10 rods south of the center; what is the distance from the north side to the summit if the hill is 198 feet high?

22. What is the distance around the park?

23. How much less is the distance around the hill than the distance around the quarter that it occupies?

24. If railroad ties cost 28 cents apiece, what would the ties in this railway cost if laid $2\frac{1}{2}$ feet apart from center to center?

25. In what direction does the city seem to be growing?

26. What is the length of the race track?

27. How much land is enclosed by it?

NOTE: The student will readily observe that the possibilities of this diagram are far from being exhausted. The author has seen fit to omit a number of good problems, and leave it for the student to make at least 20 more. It will be found to be a valuable exercise. Work the problems.

LESSON XIX.

MISCELLANEOUS PROBLEMS.

1. Fifty acres of land are fenced off into two square fields, the area of one of which being four times as much as the other. At 60 cents a rod, what is the cost of fencing both fields if they are not adjacent?

2. How many feet of lumber in the following:

24 2 x 10 joists 18 feet long.
4 beams 20 feet long and 9 inches square.
200 pieces of siding 12 feet long and 4 inches wide.

3. How much more will it cost, at $1.50 a rod, to fence 10 acres in the form of a square, than in the form of a circle?

4. The diagonal of a square is 8, what is the length of a side of the square?

5. At $1.25 a rod, what is the cost of fencing a square field whose area equals that of another field, whose parallel sides are respectively 80 rods and 120 rods, and the distance between them 30 rods?

6. Find the area of a rectangle 24 x 20. Find the area of another, half of these dimensions.

7. A box, without a lid, is 5 feet long, 4 feet wide and 3 feet deep, inside dimensions. How many square feet of zinc will it take to line the bottom and sides of this box?

8. Find the area of a triangle whose longest side is 25 rods and whose altitude is 5 feet.

9. Find the cost of the boards of a sidewalk 4 feet wide, around the outside of a garden 200 yards by 90 feet. The boards cost $22 per M, and are 1½ inches thick.

10. Find the cost of a like walk around the inside of the garden.

11. If 4 x 5 rafters are used, and they are placed 30 inches apart, how many feet of lumber are there in the 20 foot rafters of a double roof 40 feet long?

12. A man mowing, walks .4 miles an hour. It takes him 72 minutes to mow a plot of 1056 square yards. How broad a swath does he mow?

13. A triangle contains 233⅓ square yards. The perpendicular distance from the apex to the base is 20 feet. Find the base.

14. Draw a circle 4 inches in diameter. Draw two of its diameters, the one at right angles to the other. Draw the lines connecting the points where the extremities of the diameters meet the circumference. What figure is inscribed in the circle?

15. To find its area, what dimensions must you know?

16. What is the base? The altitude?

17. What is the area?

18. Draw another circle 6 inches in diameter and find the area of the inscribed square.

19. Draw an inscribed square on the small end of a log sawed off. Suppose the diameter to be 24 inches, what is the area of the square?

20. If this log is 16 feet long, and the slabs are cut off leaving only a square piece of timber, how much lumber does it contain? The number of feet of lumber can safely be figured in this way.

21. How much lumber in a log 18 feet long 4 feet in diameter?

22. A log 20 inches in diameter and 20 feet long?

23. 50 inches in diameter and 16 feet long?

24. A log 12 feet long and 8 inches in diameter?

25. Find the exact number of inches and feet of inch lumber needed for a box with cover, 4 x 5 x 6 feet on the outside.

26. What are its dimensions on the inside?

27. How much lumber in a trough made of 2 inch planks? It is 16 feet long, 2 feet high, the bottom is 8 inches and top 22 inches wide, outside dimensions.

28. A garden 40 rods long is laid out with semi-circular ends; it is 10 rods wide. Find area.

29. Find area of a 4 foot path around the outside of it.

30. Around the inside of it.

31. How much is gained by crossing a 165 x 532 yard field diagonally, instead of going around?

32. A street 5 miles long contains 30 10-33 acres; find width.

33. The area of a trapezoid is $2\frac{1}{2}$ acres, and the sum of the parallel sides is 40 rods. Find altitude.

34. A rope 73 feet long reaches from the edge of a ring to the top of a center pole 55 feet high. How wide is the ring?

35. Find the circumference of the ring.

36. Find the area of the circle.

37. A stick of timber 3 feet square and 16 feet long is sawed into inch lumber. How much lumber in it, 1-6 being lost in sawing?

38. Which will carry the largest amount of water, two 3-inch or one 4-inch tile? Consider the speed the same in both cases.

39. Find the difference in area of a square and an equilateral (*equal-sided*) triangle, each of whose sides is 6 feet.

40. Two circles with a radius of 7 feet each are placed in a circle with a radius of 14 feet. Find the area not covered.

41. How many pickets 3 inches wide will it take to put up a fence 40 rods long, 3 inches being allowed between pickets?

42. Suppose this fence has to run over a hill 40 feet high, how many more pickets will it take?

43. A quarter section is divided into square 10 acre fields. Find the number of rails at 14 to a rod necessary to fence it.

44. Find cost of paving a hexagonal court, each side being 60 feet, at a cost of $3 per square yard.

45. A pole whose circumference is 9 inches and height is 30 feet has around it a wire in the form of a spiral which goes around it once every foot. Find length of wire. (Hypotenuse of a triangle.)

MENSURATION OF SOLIDS.

LESSON XX.

PRISMS AND CYLINDERS.

Take a common crayon box, and you have a good example of a prism. Set it on end; its height in that position we'll call its altitude, the rectangle on which it stands is its base, and the top rectangle is its other base. You will notice that the two bases are equal; this is true in all prisms. As long as the two bases are equal, and have the same shape and are parallel, the solid is a prism. If the bases are squares it is a square prism; if they are triangles, a triangular prism; if a rectangle, rectangular prism. If the bases are circular the solid is a cylinder.

1. Back to the crayon box. What is the area of its base?

2. What is the area of its sides?

3. Find the area of the entire surface of the crayon box.

4. What is the surface of a box whose base is 4 x 6 feet and whose altitude is 10 feet?

5. Take a triangular prism. The base of the triangle which forms this base of the prism is 6, altitude 8. Find the area of both bases?

6. What dimensions do you want, to find the area of the lateral sides? Cut a triangular prism from a piece of pine and measure.

7. Find the area of the entire surface of this prism.

8. If the bases are circles having a diameter of 4, what is the area of the bases of this cylinder?

9. Take a fruit can. Measure its diameter and find the area of its bases. Now cut a piece of paper, as wide as the can is high, and wrap it tightly around the can once. Cut off the paper not needed. Now take the paper off and hold it flat.

10. What figure have you?

11. Find its area.

12. To what does the base in that figure correspond in the cylinder?

13. To what does the altitude correspond?

14. What, then, is the entire surface of the fruit can?

15. Take another cylinder 10 inches in diameter and 20 inches high. Find its entire surface.

You have learned that the circumference of the cylinder corresponds to the base of a rectangle, and the altitude of the cylinder to the altitude of the rectangle. Hence the

Curved Surface of Cylinder = Circum. × Altitude.

In like manner the lateral surface of any prism may be found by multiplying the perimeter, or the distance around the base, by the altitude. In the preceding chapter you have learned how to find the area of any shaped base.

16. Find the surface of a cylinder whose diameter is 23 and whose altitude is 10.

17. Radius 20, altitude 100.

18. Circumference 157.08 and altitude 100.

19. Area of the base is 31416 and altitude 1000.

20. Find the surface of a rectangular prism whose base is 8 x 9, and altitude 40.

21. Of a triangular prism, the base of whose base is 8 and altitude 6, and altitude of prism 9.

22. Of a square prism, one side being 7 and altitude 24.

23. Take a box 4 inches long, 3 inches wide and 5 inches high. Place one row of inch cubes along the length inside of the box. How many cubes in the row?

24. How many such rows can you place on the bottom of that box?

25. How many such layers to fill the box?

26. How many cubic inches in the box?

1 cubic inch $\times 4 = 4$ cubic inches; $\times 3 = 12$ cubic inches; $\times 5 = 60$ cubic inches. For all practical purposes it is enough to say, the area of the base times the altitude equals the solid contents. The area of the base in this case is 12, and that times the altitude equals 60.

Contents of Prism = Area of Base × A.

27. The base of a prism is 5 square feet, the altitude is 26 feet. Find the contents.

28. The area of the base of a triangular prism is 24 and altitude 30. Find the contents.

29. Find the volume of a hexagonal prism, the area of whose base is 6, and whose altitude is 10.

30. Find the volume of a cylinder, the area of whose base is 12, and whose altitude is 20.

31. The base of a rectangular prism is 5 x 8 feet. The altitude is 14 feet. Find the volume.

32. One side of the base of a triangular prism is 12, the altitude of the base is 10, and the altitude of the prism is 20. Find the volume.

33. The radius of the base of a cylinder is 10, the altitude is 100. Find the volume.

34. What is the surface of this cylinder?

LESSON XXI.

PROBLEMS.

1. The circumference of a cylinder is 31.416, the altitude is 20; find the volume.

2. Find the surface.

3. Find the entire surface of a prism whose base is an equilateral triangle, the perimeter being 18 feet, and the altitude 15 feet.

4. What is the volume of a triangular prism whose altitude is 15 feet, and the length of each side of whose base is 20 feet?

5. Find the area of its surface.

6. What is the volume of a solid 6 feet 3 inches long, 4 feet 6 inches wide and 8 feet high?

7. Its surface.

8. Find the curved surface of a piece of stove-pipe 6 inches in diameter and 2 feet long.

9. The volume of this cylinder.

10. A hexagonal prism is 12 inches high with 2 inch sides; find the entire surface.

11. Its volume.

12. How many cubic yards of earth will be removed by digging a cellar 12 feet long, 10 feet wide and 2 2-5 feet deep?

13. How many cubic feet of air will a room 14 feet long, 12 feet wide and 10 feet high contain?

14. Find the number of square feet in the walls, ceiling and floor.

15. What will it cost to dig a cellar 16 feet long, 12 feet wide and $3\frac{3}{4}$ feet deep, at 75 cents per cubic yard?

16. If a block of stone 5 inches long, 3 inches wide and 2 inches high weighs 44 ounces, what will be the weight in pounds of a cubical block of stone whose edge is $2\frac{1}{2}$ feet?

17. What is the volume of a 6 x 8 foot rectangular prism whose altitude is 20 feet?

18. Find its surface.

19. What is the volume of a cylinder whose length is 16 feet and the area of whose base is 54 square feet?

20. Find its surface.

21. What is the volume of a cylinder whose altitude is 60 feet and whose diameter is 10 feet?

22. Find its surface.

23. If one cubic foot of water weighs $62\frac{1}{2}$ pounds, what is the weight of water sufficient to fill a hollow square prism 18 feet long, and whose base is 5 feet wide?

24. Required the solid contents of cylinder whose altitude is 15 feet and its radius 1 foot 3 inches.

25. Find its surface.

26. What weight of water will a rectangular tank contain, the length being 8 feet, breadth $5\frac{1}{2}$ feet and depth 7 feet?

27. What is the value of a pile of tan-bark 120 feet long, 36 feet wide and 12 feet high, at $4.50 a cord?

28. A sleigh upon which 4-foot wood is piled is 12 feet long. How high must the wood be piled to contain $1\frac{1}{2}$ cords?

29. What is the volume of a triangular prism whose length is 12 feet, and each side of whose base is 3 feet?

30. Find its surface.

31. How many gallons of water will a cylindrical boiler contain if 25 inches high and 12 inches in diameter?

32. Find how much tin it takes to make such a boiler.

33. Take an ordinary tumbler whose bottom is smaller than the top; fit a piece of paper around it. Can you find the area of it?

34. Recall the trapezoid and find the area.

35. The circumference of the bottom + the circumference of the top ÷ 2 = ?

36. Find the entire surface of tumbler.

37. What is the entire surface of a pail the diameter of whose bottom is 12 inches, and top 20 inches, the slant height is 20 inches?

38. How much tin in a pail the area of whose base is 78.54 square inches, the cover 113.0976 square inches, the slant height is 20 inches?

39. Find the area of an inch rim on the cover.

40. Find how much water the pail will hold, if altitude is 19 inches.

41. How much tin in the pail at your school?

42. Find how much water it will hold. Prove by measuring.

43. How much sheet iron in a stove-pipe 6 inches in diameter and 14 feet long?

44. Find the contents.

45. How much sheet iron does it take for the stove-pipe in your school?

46. Find its contents.

LESSON XXII.

PYRAMIDS AND CONES.

1. From a large potato cut a pyramid with a square base.

2. Look at it a while and tell how to find the area of its base and sides.

3. Find the surface of the ones you have.

4. Find the surface of a pyramid whose base is 5 x 6 inches and slant height is 10 inches.

5. Find the surface of a triangular pyramid the altitude of whose base is 6, one side 12, and the slant height is 20.

6. Cut an equilateral triangular prism from a large potato.

7. Can you cut that into exactly three pyramids with the same base and altitude as that of the prism? It can be done.

8. Then, if a triangular prism can be divided into three equal pyramids having the same base and altitude that the prism has, the pyramid is what part of this prism?

$$Volume\ of\ Prism = B. \times A., \qquad \text{then,}$$
$$Volume\ of\ Pyramid = \tfrac{1}{3}\, B. \times A.$$

9. What is the volume of a pyramid whose base is 24 and altitude 60?

10. Base 86 and altitude 288?

11. What is the volume of a cone whose base is 8 and altitude 27? (You have by this time learned that a cone is nothing more or less than a pyramid with a infinite number of sides.)

12. Find the volume of a cone whose base is 22 and altitude 66.

13. Base 7 feet and altitude 22 inches.

14. Find the volume of a cone whose base diameter is 6 and whose slant height is 5. Distinguish carefully between slant height and altitude. The slant height is the distance from the circumference of the base to the apex, and the altitude is the perpendicular distance from the base to the apex.

15. Take a cone. Wrap a piece of paper around its curved surface so that it fits exactly.

16. Take the paper off and tell what figure you have.

17. It is part of a circle, and can be divided into an infinite number of triangles. How would you find the area of it?

18. Find the area of the figure you have.

19. Find the surface of a cone the circumference of whose base is 16 and slant height 24.

20. Radius 24 and slant height 26.

21. Find the surface of the great pyramid 764 feet square, and having a slant height of 451 feet.

22. Find its volume.

23. How many cords of stone are there in a pile 24 feet long, 16 feet wide, and 3 feet high? (In estimating the amount of stone for a wall one cord makes 100 cubic feet of wall, and no smaller part than a quarter cord is allowed; it is customary to measure around the outside of the wall in estimating.)

24. How many cubic feet of wall can be laid with $7\frac{1}{2}$ cords?

25. How many cords of stone will build a wall 200 feet long, 12 feet high and 3 feet thick?

26. How many cords of stone are required for a cellar 26 x 16 feet and 9 feet high, if the walls are 2 feet thick?

27. Build a 6 foot high and 2 foot thick foundation under the second plan in this book, and estimate the amount of stone.

28. How many cords of stone in the foundation of your school?

LESSON XXIII.

SPHERES.

1. The volume of a pyramid = ?

2. If the base of a pyramid is 6 and the altitude 10, what is its volume?

3. If the sum of the bases of a number of pyramids is 30 and the altitude 10, what is the volume of all?

4. If you can have, or can make, a number of pyramids of equal base and altitude, the bases as small as possible, and put them together with their apexes at one point, the solid you have resembles what?

5. Suppose that the sum of the bases of all these pyramids that you put together is 1256 and the altitude of each is 10, what is the volume of the solid?

6. By this time it must have occurred to you that a sphere is composed of an infinite number of pyramids; that the bases of the pyramids correspond to the surface of the sphere, and the altitude of the pyramid to the radius of the sphere.

7. The volume of a sphere, then, equals $\frac{1}{3}$ of the radius times the area of the surface.

$$Vol.\ of\ Sphere = \tfrac{1}{3}\, r \times Surface.$$

8. Now follows something that must be remembered. In the circle you had to remember that the circumference equals 3.1416 times the diameter; in the sphere you must remember that the *surface* equals 3.1416 times the *square* of the diameter.

$$Circumference\ of\ Circle = 3.1416 \times D.$$
$$Surface\ of\ Sphere = 3.1416 \times D^2.$$

Learn this thoroughly, and then study the following:

D equals $2\,r$; you see that. Then D squared equals $(2\,r)$ squared. Then, $Surface\ of\ Sphere = 3.1416 \times (2\,r)^2$. See that?

Above you learned that

$$Volume\ of\ Sphere = \tfrac{1}{3}\, r \text{ times the } Surface.$$

Now, instead of writing *Surface* let us put into this equation the $3.1416 \times (2\,r)^2$, which equals the surface, and we have

$$Vol.\ of\ Sphere = \tfrac{1}{3}\, r \times 3.1416 \times (2\,r)^2.$$

Let us make that smaller by joining the r's. The square of 2 is 4; the square of r is r^2; $\frac{1}{3}$ times 4 is 4-thirds; r times r^2 is r^3; together, 4-thirds r^3. Then

$$Volume\ of\ Sphere = \text{4-thirds } r^3 \times 3.1416.$$

9. If the radius is 12, what is the volume of the sphere? 4-thirds $\times (12)^3 =$ 4-thirds of 1728; this $\times\ 3.1416 = ?$

10. Radius 3, what is the volume?

11. What is the surface?

12. Radius 6; find volume.

13. Find surface.

14. Diameter 24; find volume.

15. Find surface.

16. If volume $=$ 4-thirds $r^3 \times 3.1416$, then 4-thirds $r^3 =$ Volume $\div$ 3.1416, and

$$r^3 = \frac{\text{Vol.} \div 3.1416}{\text{4-thirds}} \quad \text{or}$$

$$\text{Vol.} \div 3.1416 \times \tfrac{3}{4}; \qquad \text{or} \qquad \tfrac{3}{4}\,\text{Vol.} \div 3.1416.$$

$$r = \sqrt[3]{(\tfrac{3}{4}\,\text{Vol.} \div 3.1416)}; \quad \text{or, in words,}$$

if the volume is given, divide $\frac{3}{4}$ of it by 3.1416, and extract the cube root of it; the result is the radius.

18. If the volume is 113.0976, what is the radius?

19. Find the surface.

20. Volume is 12866.9936; find radius.

21. Surface.

22. Volume is 4.1888; find radius.

23. Surface.

24. Radius is 21, find surface.

25. Find volume.

26. A 3 inch circle is placed within a 4 inch circle, how much of the larger circle is not taken up?

27. A circular sheet of gold 20 inches in diameter is to be divided equally between two children, one of whom is to receive his in the form of a circle, and the other in the form of a ring around the circle. What is the diameter of the circle and the width of the ring?

28. Find the cost of painting a church spire, at 25 cents a square yard, whose base is a hexagon 5 feet on each side, slant height 60 feet.

29. Find the solid contents of a cone, the diameter of whose base is 6 feet and its altitude $10\frac{1}{2}$ feet.

30. Find the solid contents of a cone whose altitude is 24 feet, and the diameter of whose base is 30 inches.

31. What is the cost of a triangular pyramid of marble, whose altitude is 9 feet, each side of the base being 3 feet, at $\$2\frac{1}{2}$ per cubic foot?

32. Find its surface.

LESSON XXIV.

DRILL EXAMPLES.

Find the missing dimensions.

RECTANGLES.

Base.	*Altitude.*	*Area.*	*Diagonal.*	*Distance Around.*
9	12	...	...	...
..	16	256	...	...
..	15	...	25	...
8	6	...	...	...
15	...	540	...	...
..	24	432	...	...
..	5	...	...	13
6	9	...	...	...
..	12	96	...	...
9	...	315	...	...
10	...	...	...	30
..	...	144	...	...
$\frac{1}{2}$	...	72	...	...
9	$\frac{2}{3}$	...	...	...
$\frac{5}{8}$	...	$\frac{3}{4}$	...	...
..	...	256	...	...
..	$\frac{1}{4}$	...	...	$\frac{7}{8}$
.05	...	1	...	...
$1\frac{1}{2}$ ft.	...	24 in.	...	...
4 in.	...	...	...	9 ft.
..	4.2	80	...	...

RIGHT TRIANGLES.

Base.	*Altitude.*	*Area.*	*Hypotenuse.*	*Distance Around.*
8	10	...	...	...
..	32	192	...	...
..	20	...	25	...
40	30	...	...	120
..	24	384	...	99
9	8	...	...	...
40	...	...	50	...
$\frac{3}{4}$	$\frac{2}{3}$	...	...	...
5	...	90	...	...
..	8	...	10	36
..	.01	.05	...	...
..	40	840	...	...

TRAPEZOIDS.

1st Base	*2nd Base.*	*Altitude.*	*Area.*
9	8	12	..
..	6	5	25
12	..	30	..
9	10	..	90
$\frac{1}{2}$	$\frac{3}{4}$	.05	..

CIRCLES.

Radius.	*Diameter.*	*Circum.*	*Area.*
12	..	....	....
..	20	....	....
..	..	25.1328	....
..	..	....	254.4696
$\frac{1}{6}$	..	....	....
..	$\frac{2}{3}$	....	....
..	..	.31416	....
..	..	....	3.1416
..	.005	....	....
..	..	....	1

RECTANGULAR PRISMS.

Base.	*Altitude.*	*Dim. of Base.*	*Volume.*
16	40	....	....
..	16	4 x 12	....
..	..	7 x 9	441
$\frac{3}{4}$	$\frac{7}{8}$	....	....
.05	..	....	.08
..	.6	$\frac{3}{4}$ x $\frac{3}{8}$	....

CYLINDERS.

Alt.	*Base.*	*Diam.*	*Circumference.*	*Entire Surface.*	*Vol.*
10	.....	12	......	......	......
40	.....	..	157.08	......	......
10	78.54	..	......	......	......
5	.....	..	......	......	90

PYRAMIDS.

Base.	*Alt.*	*Vol.*	*Dim. of Base.*	*Slant Height.*
5	7	..	..	..
..	8	56	..	..
9	..	24	..	..
..	..	..	4 x 4	10
..	10	..	5 x 5	..
..	..	90	8 x 8	..

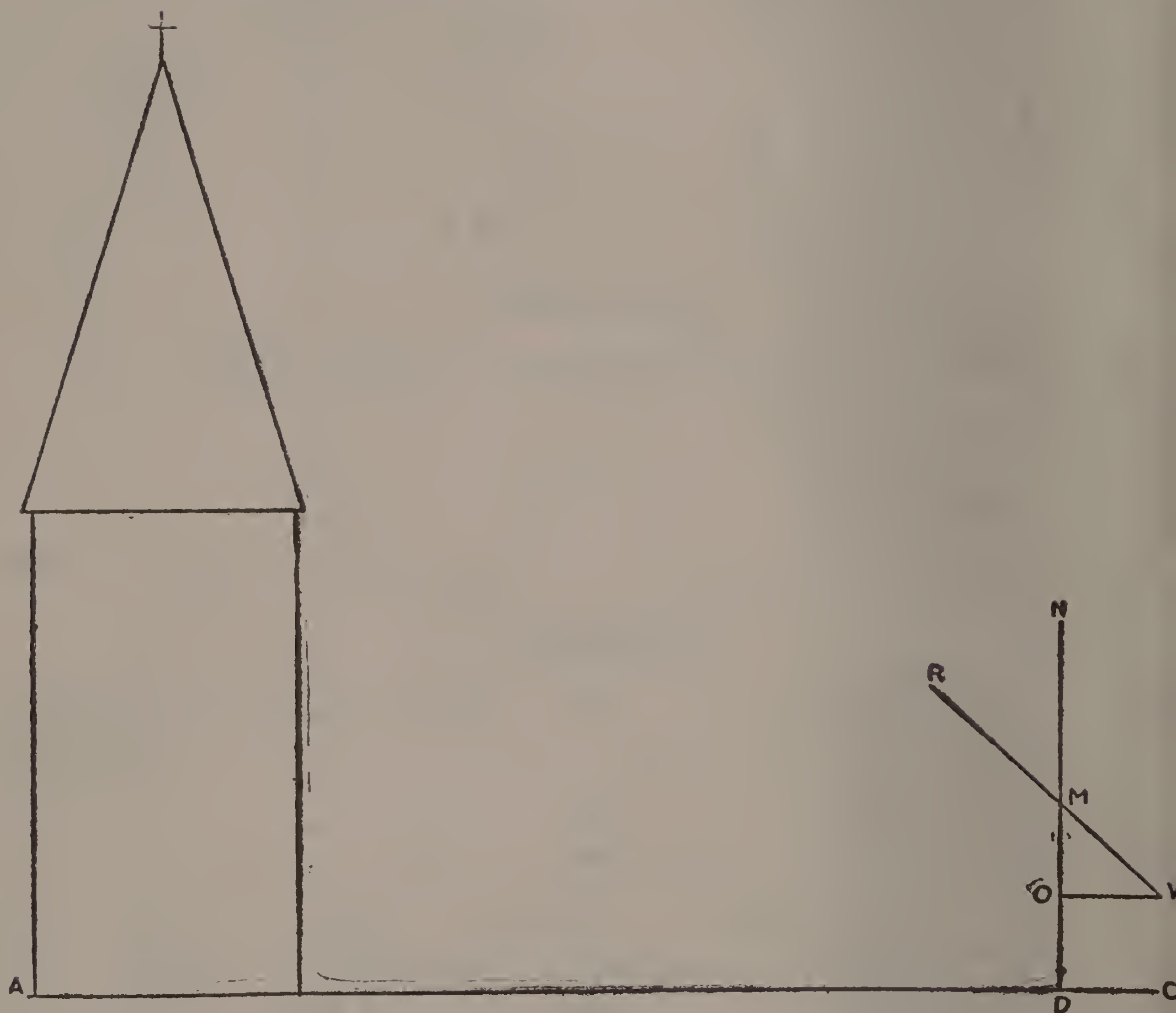
SPHERES.

Radius.	*Circumference.*	*Surface.*	*Volume.*
6	......	....	....
..	9.4248	....	....
3	......	....	....
..	......	244.4696	....
..	......	....	1325.3625
..	1.5708	....	....

LESSON XXV.

MISCELLANEOUS PROBLEMS.

HOW TO MEASURE HEIGHT OF OBJECTS.



Make a pole about 8 feet in length, *n d*. At right angles to this and 4 feet from one end firmly fasten a lath about 3 feet long, *o v*. To the end of this lath at *v* join, with a bolt, a longer lath, *v r*, in such a manner that the long lath slides up and down the pole *n d*. Set this instrument any convenient level distance, say 50 feet from the object the height which is to be measured, and placing the eye at *v* sight the long lath *v r* by sliding it up and down so that it points to the top of the object to be measured. Now hold *v r* in place and measure the distance from *m* to *o*. Let us say that it is

$2\frac{1}{2}$ ft. From v to the object is 54 ft. The proportion in all problems of this kind is: $o\,v : c\,a :: m\,o :$ x, or the height of the object. $3 : 54 :: 2\frac{1}{2} : x = 45$; add to this the 4 feet, because the eye is 4 feet from the ground, $= 49$ feet.

1. If a spire is 60 feet from the eye and $o\ m$ is 3 feet, how high is it?

2. If $o\ m$ is $3\frac{1}{2}$ feet?

3. Distance 80 feet and $o\ m$ $2\frac{1}{3}$ feet, how high?

4. Find the height of the school, the church, and the height of trees.

5. The diameter of the earth is 7,920 miles. What is the area of its surface?

6. On a 12 inch school globe, each square inch represents how many square miles of surface?

7. How much of the globe's surface represents Wisconsin?

8. If a cannon ball weighs 22 pounds, what is the weight of another with a diameter twice as great?

9. A 12 inch solid iron ball is melted and run into the form of a 3 inch cylinder. How long is the cylinder?

10. In cutting a foot sphere from a foot cube, what fraction of the cube is cut away?

11. Four pipes, each 2 inches in diameter, empty into a tank. What must be the diameter of a single pipe to carry away all of the water?

ANSWERS.

Lesson IV.

8. 270.

Lesson V.

1. 16; 32; 20; 24; 28; 16.
2. 15; 21; 12.
3. 30; 42; 24; 15; 21; 12; 22½; 31½; 18; 33¾; 44¾; 27.
4. 8; 16.
5. 5; 10; 12; 23⅓; 26⅔; 42; 36; 4⅔; 19¼; 24; 17½; 4⅔; 40; 37½; 37⅓; 12; 74⅔; 10⅔; 56; 66⅔; 240.
6. 16 sq. ft.
7. 160 sq. ft.
8. 160; 160; no.

Lesson VI.

1. 192; 192; 192.
2. 200; 240.
3. 10 x 20; 10 x 16.
4. 4; 200; 160.
5. 810; 450; 450.
8. 264; 528.
9. 22.
10. 29.
11. 540; 720.
12. $30.
13. $4; 36¢.
14. $9.50.
15. $5.04.
16. $4.80.
17. $77.
18. $15.77.

Lesson VII.

1. Boards, $184.32.
   Shingles, $150.80.
   Planks, $32.00.
   Total, $367.12.
2. $40.59.
3. $18.40.
4. 10 ft.; 12 ft.; 12 ft.
5. $35.88; 14 ft.
6 $109.22.
7. $75.64.
8. $159.76.
9. 16 ft.

Lesson VIII.

1. $16.
2. $35.12+.
3. $132.
4. $10.34.
6. $22.85.
7. $8.64.
8. $17.
9. $47.60.
10. 12.
11. 180 ft.; 8 d. rolls; 6 d. rolls.
12. $1.98.
13. $2.80.

Lesson IX.

1. $23.34.
2. $43.52.
3. $53.85.
4. $99.60.
5. $7.08.
6. $46.80.
7. $25.76; 16 ft.
8. $6.40.
9. $19.80.
10. $35.20.
11. $77.
12. $8.50.
13. $6.92; 18.43; 3.69.
14. $23.26.
15. $15.02.
16. $1.60; $3.40.
17. $5.49; $7.04.
18. $56.
19. $7.33.
20. $23.96.
21. $3.96.

Lesson XI.

8. 30.
9. 110; 400.
13. 24.
14. 12½.
15. 35½.
16. 73½.
17. 256.

Lesson XII.

3. 20.
4. 100.
5. 215.
13. *a* 16; *b* 26; *c* 24; *d* 18; *e* 21; *f* 20; *g* 9; *h* 12; *i* 8; *j* 48; *k* 45; *l* 30; *m* 1+; *n* 30; *o* 20; *p* 75; *q* 48; *r* 120; *s* 12; *t* 111.
14. 48.
16. 22+.
17. 28+.
18. 14+.

Lesson XIII.

4. 20; 7.
8. 8.
9. 10.
10. 150.
11. $528.
12. 64.26+ rds.
13. 13; 120.
14. 336 ft.
15. 350.
16. 160.
17. 128 rds.
18. 40 ft.
19. 960 sq. ft.
20. 1540 sq. ft.
21. 250 sq. ft.
22. 520 sq. ft.
23. 128 ft.
24. 200 ft.
25. 70+ ft.
26. 14 ft., nearly.
27. 13 ft.
28. 5 ft.
29. 46+ft.
30. 58+ft.
31. 20.
32. $27.50.
33. 67 ft.; 76.
34. 12 ft.; 10 or 20 ft.
35. $50.
36. $19.
37. $1.90.
38. $10.
39. 1536.
40. 384.
41. 10 ft.
42. 587.
43. 224.
44. $10.91.
45. $6.16.
46. $6.33.
47. 16.13.
48. 21; 42.
49. 18 ft.
50. 504.
51. 88.
52. $24.96.
53. $4.32.
54. $24.19.
55. $54.90.
56. $33.60.
57. $69.89.
58. $12.10.
59. $65.67.
60. $139.
61. $12.96.
62. $6.48.
63. $7.56.
64. $10.80.
65. $3.60.
66. 68.
67. 44.
68. 78.
69. 156.
70. 247.
71. $5.76.
72. $22.92.
73. $4.08.
74. $5.80.
75. 141.
76. 3.
77. 69¢.
78. $1.60.
79. 45¢.
80. $53.33.
81. $1.60.
82. $7.20.
83. 80¢.
84. $6.67.
85. 24 +.
86. 26.83 + ft.
87. 60.88 + ft.
88. Rectangle.
89. 221.28 + ft.
90. 24 + ft.
91. Nearly 6 ft.
92. 1152 sq. ft.
93. 12 x 20.
94. 12.65 ft.
95. 15 + ft.
96. 70 + ft.
97. Trapezoid.
98. 264 sq. ft.
99. $1\frac{1}{8}$ ft.
100. 42 ft.

Lesson XIV.

15. 15.708.
16. 100.531.
17. 5.

Lesson XIV.—Con.

18. 4.
19. 18.
20. 201.0619.
21. 1963.4954.
22. 100.531.
23. 6.
24. 113.0973.
25. 314.1593.
26. 254.469.

Lesson XV.

2. 7.
3. 14; 43.9823.
4. 41.
5. 50.2655.
6. 15.708.
7. $70.69; 490.8739.
8. 20.73 + A; 204.2035 rds.
9. 125.6637.
10. 1256.6371.
11. 12.5664.
12. 63.6173.
13. $12.36 less.
14. 25.
15. 7 near enough.
16. 38.4845.
17. 21.9911.
18. 3661; about 5½.
19. 19.635.
20. 15.708.
21. 4.9087; 3.927.
22. 1.3412.
23. 2.4523.
24. 1.9635.
25. 4.9087; 7.854.
26. 2.4543.
27. 4.908; 5.8905.

Lesson XVI.

1. 3⅓ ft.
2. $3540.
3. 15840.
4. 42½ about.
5. 585.6.
6. About .6 mi.
7. 14520.
8. 9.
9. $12.
10. 10⅝.
11. 6¾.
12. 124⅔.
13. 64 rds.
14. 150 rds.
15. 2827.4334 sq. ft.
16. 648.
17. $30.
18. 9600 sq. ft.
19. 144.44 sq. yd.
20. 169.
21. 10.9956.
23. $80,000.
24. $70,000.
26. $14,000.
27. 3520.
28 79 + ft.
29. 20.
30. 3926.9908.
32. 61.
33. 50.
34. 360 A.
35. 608.
37. 24899.
38. 94.2478.
39. 20 rds.
40. $18⅓.
42. $66.77.
43. $1170.
44. 40 yds.
45. ⅝.
46. 25 ft.
47. 52 ft.
48. 46⅔.
49. $6.48.
50. 36.19 +.
51. 255 +.
52. 37.6991.
58. 7 1-5.
59. 99.

Lesson XVII.

1. 640 A.
2. 160 A.
6. 80 A.
7. S. ½ of S. E. ¼.
8. 80 A.
10. 40 A.
11. S. E. ¼ of N. E. ¼; 40 A.
13. N. W. ¼ of N. E. 10 A.
    N. E. ¼ of N. E. ¼ of N. E. ¼; 10 A.
    ¼ of N. E. ¼;
14. S. ½ of S. W. ¼ of N. E. ¼; 20 A.
15. N. E. ¼; 10 A.
16. 5 A.
17. S. ½ of N. W. ¼ of S. W. ¼ of N. E. ¼; 5 A.
18. 20 A; 100 A.
19. 140 A; 140 A.
20. 95 A; 280 A; 320 A; 70 A.
21. 80 A; 40 A.
22. 10 A.

Lesson XVII.—Con.

23. 5 A.
24. 8.
25. 16.
26. 320.
27. 40 rds.; 80 rds.
28. 320 rds.
29. 120 rds.
30. 480 rds.
31. 280 rds.
32. 240 rds.
33. 120 rds.
34. 320 rds.; 480 rds; 640 rds.
35. d h j
36. 200 rds.; 280 rds.
37. 1-8; 1-32; 1-16; 1-64; 1-128.

Lesson XVIII.

2. 40 A; 20 A; 30 A; 10 A; 5 A; 5 A.
3. 200 A.
4. 80 A.
5. 140 A.
6. 120 A.
7. 200 A.
8. 80 A.
9. 480 A.
10. 145 A.
11. 115 A.
12. 100 A.
13. 200 A.
14. 502.656 rds.
15. 125.661 A.
16. 34.339 A.
17. About 33¼ A.
18. Nearly 33 A.; about 8½ A.
19. 1102 rds.
20. 541.80.
21. Nearly 171 rds.
22. 678 rds.
23. 137.344 rds.
24. $1219.68.
25. Southwest.
26. 411.3274 rds.
28. 71.4+ A.

Lesson XIX.

1. $288.
2. 2060.
3. About $27.
4. 5.65+.
5. About $68.75.
6. 120.
7. 74 sq. ft.
8. About 61¼.
9. $184.27.
10. 180.04.
11. 1133⅓.
12. 3¾.
13. 210.
18. 18 sq. in.
19. 288 sq. in.
20. 384 ft.
21. 1728 ft.
22. 333⅓.
23. 1666⅔.
24. 32.
25. 143 ft. 8 sq. in.
26. 3 ft. 10 x 4 ft. 10 x 5 ft. 10.
27. 151⅓ ft.
28. 378.54 sq. rds.
29. 6225.4+ sq. ft.
30. 5877.3 sq. ft.
31. 140.
32. 50 ft.
33. 15½ rds.
34. 96.
35. 301.5929.
36. 7238.2295.
37. 1440 ft.
38. 2 3-in.
39. 20.7 sq. ft.
40. About 308 sq. ft.
41. 1320.
42. Not any more.
43. 2240.
44. $311 5-9.
45. 31.32 + ft.

Lesson XX.

15. 785.3976 sq. in.
16. 1553.5172.
17. 15079.6442.
18. 19635.
19. 691152.
20. 1504.
21. 264.
22. 770.
27. 130 cu. ft.
28. 720.
29. 60.
30. 240.
31. 560 cu. ft.
32. 1200.
33. .31416.
34. 6911.5086.

Lesson XXI.

1. 1570.796.
2. 785.3996.
3. 300 sq. ft.
4. 2598 cu. ft.
5. 1246.4 sq. ft.
6. 225 cu. ft.
7. 228¼ sq. in.

Lesson XXI.—Con.

8. 452.4 sq. in.
9. 678.5856 cu. in.
10 164.78 sq. in.
11. 124.68 cu. in.
12. 10⅔ cu. yd.
13. 1680 cu. ft.
14. 520 sq. ft.
15. $20.
16 2475 lbs.
17. 960 cu. ft.
18. 656 sq. ft.
19. 864 cu. ft.
20. 525.2 sq. ft.
21. 4712.4 cu. ft.
22. 2042.04 sq. ft.
23. 28125 lbs.
24. About 75 cu. ft.
25. 127.81 sq. ft.
26. 19250 lbs.
27. $1822.50.
28. 4 ft.
29. 45 cu. ft.
30. 123 sq. ft.
31. 12.22 + gal.
32. 1168.7 sq. in.
37. 1118.4 sq. in.
38. 882.7896 sq. in.
39. 37.7 sq. in.
40. 8.29 + gal.
43. Nearly 22 sq. ft.
44. 2.74 + cu. ft.

Lesson XXII.

4. 140.
5. 238.08 +.
9. 480.
10. 8256.
11. 72.
12. 484.
13. 616 cu. in.
14. 37.6992.
19. 212.428 +.
20. Nearly 3770.
21. 1,272,824 sq. ft.
22. 46,695,680 cu. ft.
23. 11½ cd.
24. 750 cu. ft.
25. 72 cd.
26. 14 2-5 cd.
27. 15.3 + cd.

Lesson XXIII.

10. 113.0976.
11. 113.0976.
12. 893.78 +.
13. 452.3904.
14. 7238.2464.
15. 1809.5616.
18. 3.
19. 113.0976.
20. 4.
21. 201.0624.
22. 1.
23. 12.5664.
24. 5541.7824.
25. 38792.4768.
26. 14.15 +.
27. 2.93 +.
28. $25.
29. 96.9604 cu. ft.
30. 5654.8664.
31. $28⅛.
32. About 43½ sq. ft.

Lesson XXV.

1. 60 ft.
2. 70 ft.
3. 62 2-9 ft.
5. 193,919,658 sq. mi.
6. 428647.
7. About 1-9 sq. in.
8. 176 lbs.
9. 128 in.
10. .5236.
11.
12. 4 in.